Design of Experiments With MINITAB: Homework Problems (Revised April 2007)

Paul G. Mathews

Contents

Preface

The following problems are intended as homework or self-study problems to supplement *Design of Experiments with MINITAB* by Paul Mathews. The problems are organized by chapter and are intended to be solved using a calculator and statistical tables or with MINITAB or some other suitable statistical software program. The data sets given in the problem statements are already loaded into Excel and can be easily copied and pasted into MINITAB to avoid the task and risk of manual data entry. Some of the problems refer to simulations that are implemented as MINITAB macros and in Microsoft Excel spreadsheets. Before you can run the Excel macros, you will have to change Excel's Tools> Macros> Security setting to medium and restart Excel.

The problems offered here are rather minimal because I feel that any student or instructor should take the initiative to design and build his or her own experiments. You'll learn much more from those exercises than you will from doing any or all of the problems presented here. Consider doing some of the simple experiments described in the *Classroom Experiments and Labs* folder on the CD ROM - they're much harder than they look!

Many of the problems in this document refer to or make use of other documents or files contained on the CD ROM. The contents of the different folders on the CD ROM are:

- *Homework Problems* - This folder contains this file (*Homework Problems.pdf*) of homework problems, Excel files with the corresponding data sets, and other related documents and files.

- *Classroom Exercises and Labs* - This folder contains simple demonstrations and lab exercises that can be performed in a classroom, lab, kitchen, or garage.

- *Example Problem Data* - This folder contains Excel files with the data used in the example problems in the textbook.

- *Excel Design Files* - This folder contains a small collection of Excel files with some of the most commonly used experiment designs. Each experiment design worksheet has an integrated simulation macro that creates data for a fictional response. All of the design files with the same number of variables contain the same simulation function so you can compare the performance of different experiment designs effectively within the same process. These simulations, which are referred to as *sim3*, *sim4*, ... are used extensively in homework problems.

- *MINITAB 14 Macros* - This is a collection of design, analysis, and simulation macros. There are many more macros here than are described in the textbook. Open the *ReadMe.txt* file in Notepad to view a complete catalog and short description of these macros. These macros should be copied to the *Macros* folder of your *Program Files> MINITAB 14* folder so that MINITAB can find them easily.

- *MINITAB Design Files* - This folder contains a collection of MINITAB worksheets of some common experiment designs. These files are designed to be used with some older MINITAB exec simulations (e.g. *sim3.mtb*) and have been substantially superseded by MINITAB's **Stat**> **DOE** tools and the Excel design files.

1

Graphical Presentation of Data

1. Use MINITAB to construct a histogram, dotplot, stem-and-leaf plot, and boxplot of the following data:

   $$\{810, 765, 860, 825, 795, 785, 810, 790, 785, 815, 800, 790\}$$

   Add your name and the date to the graphs and the relevant Session window output and print them. Save your work in a MINITAB project file.

2. Use MINITAB's **random** function to generate a pseudo-random data set of 50 normally distributed values with $\mu = 200$ and $\sigma = 30$. To run the command from the command prompt use:

   ```
   mtb> random 50 c1;
   subc> normal 200 30.
   ```

   or use the **Calc> Random Data> Normal** menu and enter the appropriate values in the window. Create the histogram, dotplot, stem-and-leaf plot, and boxplot for these data. Make hardcopies of the worksheet, Session window, and graphs and save your work in a project file.

3. Repeat Problem 1.2 for another data set of 200 normally distributed random values with $\mu = 3.4$ and $\sigma = 0.6$ by:

   (a) Copying the appropriate commands from the Session window of Problem 1.2, editing them, and pasting them to the command prompt.

   (b) Selecting the appropriate commands from the History window and running them from the command line editor.

   (c) Saving the appropriate commands from the History window to a MINITAB *.mtb* macro file, editing the file, and running the macro.

4. Use MINITAB to generate a random sample of size $n = 200$ from a normal population with $\mu = 200$ and $\sigma = 10$. Create a histogram of the data with a superimposed normal curve and

label the axes *Frequency* and *Measurement Value*. Force the minimum and maximum values of the measurement scale to be 160 and 240, respectively, and force ticks and labels at 160, 170, ..., 240. Use arrows and tags to indicate the classes with the largest and smallest values. Title the graph *Histogram Example in MINITAB* using 18 point Arial font and make the title fit on one line.

5. Enter the following exam score data into three columns of a MINITAB worksheet and use MINITAB to construct boxplots of the exam scores by class. Use the plots to identify possible differences in the location, dispersion, and shape between the three classes.

	1	98	96	82	91	92	88	92	90	90	85
Class	2	78	74	69	65	80	79	77	74	71	75
	3	77	77	75	74	77	73	89	80	70	80

6. Stack the exam score data from Problem 1.5 into a single column of the MINITAB worksheet using the **Data**> **Stack**> **Columns** menu (or the **stack** command). Then recreate the boxplots of the exam scores using the stacked data.

7. Many DOE problems involve a response that is observed under different settings of several control variables. Despite the complexity of these problems, it is still important to show the relationship between the response and the control variables graphically. MINITAB has the ability to create matrix plots and multi-vari charts for problems involving two or more control variables.

   The useful lifetime of zinc-carbon batteries depends on the load that they drive, the duty cycle, and their lowest useful voltage called the cutoff voltage. The table below shows the operating lifetime in hours for standard zinc-carbon D-cells for loads from 8Ω to 100Ω, 100% and 17% duty cycles, and 0.8 to 1.2V cutoff voltage (M. Kaufman and Seidman, A. *Handbook of Electronics Calculations for Engineers and Technicians*, 2nd Edition, McGraw-Hill Book Company, 1988, p. 11-10).

		Cutoff Voltage (V)				
Duty Cycle (%)	Load (Ω)	0.8	0.9	1.0	1.1	1.2
100	8.0	17	11	7.2	6.0	3.2
100	25	75	51	43	38	28
100	100	430	365	320	290	240
17	8.0	20	17	15	9.2	5.1
17	25	98	89	81	70	60
17	100	430	380	360	345	310

   (a) Use MINITAB's **Graph**> **Matrix Plot** command to construct the matrix plot of the lifetime in hours, cutoff voltage, duty cycle, and load. Use the matrix plot to try to explain the relationship between the different variables.

   (b) Use MINITAB's **Stat**> **Quality Tools**> **Multi-Vari Chart** command to create a multi-vari chart of the battery lifetime as a function of the control variables. Use the multi-vari chart to explain how the lifetime depends on the other variables. You may have to consider several different charts before you find one that is easy to interpret.

8. Match each type of graphical presentation to its description.

Answer	**Presentation**
	scatter plot
	dot plot
	stem-and-leaf plot
	histogram
	multi-vari chart
	bar chart
	box-and-whisker plot
	Pareto chart

(a) Often constructed by separating the least from the most significant digits of the data.

(b) Used to prioritize different types of defects.

(c) Capable of displaying a response as a function of two or more variables, each with a limited number of levels.

(d) Uses bar lengths proportional to class frequencies and classes of equal width.

(e) A plot of one quantitative variable against another to demonstrate or test for correlation between them.

(f) Constructed from five statistics determined from the sample data set.

(g) Consists of points plotted along a number line and stacked where there are duplicates.

(h) Uses bar lengths proportional to class frequencies but classes are qualitative.

2

Descriptive Statistics

1. For the following data set:

$$\{43, 46, 54, 51, 45, 49, 42, 52, 50\}$$

use pencil and paper or a calculator to find:

(a) the median
(b) the mean
(c) the range
(d) the ϵ_i
(e) the standard deviation using the defining formula
(f) the standard deviation using the calculating formula
(g) an estimate for σ using the range

2. Use a calculator to determine the sample mean, standard deviation, and range of the following data set:

$$\{810, 765, 860, 825, 795, 785, 810, 790, 785, 815, 800, 790\}$$

Use the range to estimate the population standard deviation and compare your new estimate to the sample standard deviation.

3. Use MINITAB to check your answers from Problem 2.2.

4. Use MINITAB's **random** function (or the **Calc**> **Random Data**> **Normal** menu) to create a random standard normal data set of 1000 samples, all of size $n = 5$. Calculate the average range $\bar{R}$ from the 1000 ranges and use it to show that $d_2 \simeq \bar{R}/\sigma \simeq 2.326$ for $n = 5$.

5. Repeat Problem 2.4 to demonstrate that $d_2 \simeq 3.078$ for $n = 10$.

6. Use Table A.2 to find the following normal probabilities:

(a) $\Phi(-\infty < z < -2.44)$

(b) $\Phi(-\infty < z < 1.82)$

(c) $\Phi(1.82 < z < 2.44)$

(d) $\Phi(-1.82 < z < 2.44)$

(e) $\Phi(2.4 < x < 2.9; \mu = 2.5, \sigma = 0.8)$

(f) $\Phi(0.043 < x < 0.053; \mu = 0.050, \sigma = 0.003)$

7. Use MINITAB to confirm your answers from Problem 2.6.

8. A quality characteristic from a process is normally distributed with $\mu = 0.580$ and $\sigma = 0.008$. Find symmetric upper and lower specification limits centered at μ for x such that 99% of the parts from the process will fall within the specification limits.

9. The specification limits for a normally distributed process are $USL/LSL = 34 \pm 2$.

   (a) If the mean of the process is $\mu = 33.7$ and the standard deviation is $\sigma = 0.4$ find the fraction of the product that is in spec.

   (b) Find the new fraction defective from Part a if the process mean is centered within the specification limits.

10. Use MINITAB to plot the normal curve that has $\mu = 0.640$ and $\sigma = 0.020$. Add vertical reference lines to the plot at the specification limits $LSL = 0.590$ and $USL = 0.700$ and find the probability that x falls in this interval. Add this information to the plot.

11. A pizza can have ten different toppings and each topping can only be chosen once.

    (a) How many different pizzas can be made if the order of the toppings does not matter?

    (b) How many one-topping pizzas can be made?

    (c) How many two-topping pizzas can be made if the order of the toppings is important?

    (d) How many two-topping pizzas can be made if the order of the toppings is not important? What is the relationship between this answer and that from Part c?

    (e) Determine how many pizzas with 0, 1, ..., 10 toppings can be made and compare the total to your answer in Part a.

12. How many tests between pairs of treatment means have to be considered if there are six different treatments? Write out the list of paired comparisons to confirm your answer.

13. An experiment has two levels of each of four variables and all possible configurations of the variables are built. How many main effects, two-factor interactions, three-factor interactions, and four-factor interactions are there? How does the sum of these combinations relate to the number of levels and variables?

14. An experiment is to be performed to study three variables. The first variable will have three levels, the second will have two, and the third will have five. How many runs must the experiment have to consider every possible combination of variable levels?

15. Match each statistic to its description:

Answer	Statistic
	R
	IQR
	$\bar{x}$
	$\widetilde{x}$
	s
	x_{min}
	x_{max}
	Q_1
	Q_3

(a) The middle value in the ordered data set.

(b) An interval that contains one half of the observations.

(c) The largest value of the data set.

(d) The 75th percentile.

(e) The value, determined from the data set, above which 75% of the observations fall.

(f) An estimate of variation based on x_{min} and x_{max}.

(g) A measure of variation that takes into account all of the data values.

(h) The smallest value of the data set.

(i) A measure of location that takes into account all of the data values.

3

Inferential Statistics

1. A sample of $n = 8$ engines of a new design had an average peak shaft horsepower of $\bar{x} = 186hp$. The distribution of shaft horsepower is known to be normal with standard deviation of $\sigma = 6hp$. Write the 95% confidence interval for the population peak shaft horsepower and test the claim that the mean horsepower exceeds its target value of $\mu = 180hp$.

2. The performance of ROM chips is very sensitive to the firing temperature at a critical step in their manufacture. The firing temperature is specified to be 500C. A sample of $n = 35$ temperature readings gave an average temperature of $\bar{x} = 497C$ and standard deviation of $s = 6C$.

   (a) Is there sufficient evidence to indicate that the furnace temperature is not 500C? Use a two-tailed test with $\alpha = 0.05$. What is the p value of the test?

   (b) If there is evidence that $H_0 : \mu = 500$ must be rejected, then construct the 95% confidence interval for the true population mean temperature.

3. Each boxplot in Figure 3.1 was constructed from a random sample of size $n = 200$.

   (a) Interpret the boxplots in terms of location, dispersion, and shape.

   (b) Sketch the corresponding histograms.

   (c) Sketch the corresponding normal probability plots.

4. Hand plot the following three data sets on normal probability paper to determine if their populations are normally distributed. (You can create normal paper for manual plotting with the custom MINITAB macro *normalpaper.mac*.)

   (a) $\{83, 79, 70, 71, 73, 92, 75, 93\}$

   (b) $\{7.1, 7.1, 6.1, 7.7, 5.2, 5.6, 6.5, 6.7, 6.8, 5.9, 5.6, 6.0, 7.7, 6.1\}$

   (c) $\{14, 18, 40, 44, 47, 57, 72, 73, 86, 87, 87, 89, 97, 101, 101, 111, 166, 205, 304, 516\}$

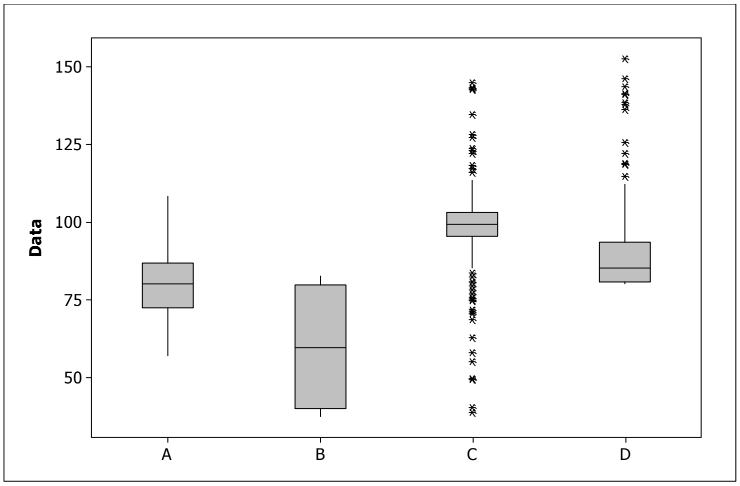

FIGURE 3.1. Boxplots of samples from four populations.

5. Write a MINITAB exec macro that creates a normal probability plot of a random normal data set of specified sample size. Run the macro twenty times for samples of size $n = 20$ and interpret each normal plot. Pay special attention to those cases that appear to deviate substantially from normality. What does the frequency of such cases suggest about the interpretation of normal plots of such small sample size?

6. The exact probability plotting positions p_i for normal plots are given by the condition:

$$b\,(i-1; n-1, p_i) = 0.5$$

where $b\,(x; n, p)$ is the binomial probability:

$$b\,(x; n, p) = \binom{n}{x} p^x\,(1-p)^{n-x}$$

Construct and compare the probability plots using the exact p_i and those determined using the approximate methods of Benard and the mid-band percentiles for the data from Example 3.24. Why are the approximations tolerated instead of using the exact probability plotting positions?

7. Random samples from two processes yield $n_1 = 9$, $\bar{x}_1 = 770$, $s_1 = 17$ and $n_2 = 12$, $\bar{x}_2 = 781$, $s_2 = 26$. The distributions of x_1 and x_2 are expected to be normal. Test to see if there is evidence that the two processes have equal variation, then perform an appropriate test for a difference in location. Use $\alpha = 0.05$ for both tests.

8. A critical dimension on a pressure fitting is intended to be $0.500in$. If the parts run larger they can become difficult or impossible to assemble. If the parts run smaller there is a chance that they will leak. A random sample of $n = 18$ parts was drawn from a large lot and inspected. The observations were:

$$x = \{0.497, 0.494, 0.496, 0.499, 0.499, 0.499, 0.497, 0.491, 0.495, 0.501, 0.504, 0.501, 0.492, 0.497, 0.500, 0.498, 0.498, 0.497\}$$

(a) Test the hypotheses $H_0 : \mu = 0.500$ vs. $H_A : \mu \neq 0.500$ at $\alpha = 0.05$.

(b) Is the normality assumption satisfied?

(c) Construct and interpret the 95% confidence interval for the true population mean.

9. Reflective tape used to accent outdoor clothing like running shoes and jackets is made by laminating tiny reflective glass beads between mylar plys. Some beads can be damaged in the laminating process which decreases the reflectance of the finished product. An experiment was performed to compare the reflectance of the standard laminating process to a proposed process that could be run at a faster production rate. The same raw materials (beads, mylar, and glue) were run through both processes. Random samples made using the standard (1) and proposed (2) process delivered the following reflectance in percent:

$$x_1 = \{94.0, 90.9, 87.6, 94.5, 90.5, 92.6, 93.1, 90.9, 89.9, 92.4, 95.1, 89.6, 89.8, 88.9\}$$
$$x_2 = \{92.4, 97.0, 90.6, 93.4, 88.6, 93.6, 87.1, 91.1, 92.0, 91.1, 93.9\}$$

(a) Test the hypotheses $H_0 : \mu_1 = \mu_2$ vs. an appropriate alternative hypothesis at $\alpha = 0.05$.

(b) Are the normality and homoscedasticity assumptions satisfied?

(c) Construct and interpret the appropriate 95% confidence interval for the true difference between the population means.

10. The pressure required to open check valves for handling gases and liquids was compared for valves provided by two manufacturers. A lower opening pressure is desired. The pressures in Pascals observed for random samples taken from the two manufacturers are:

$$x_1 = \{10581, 10087, 15439, 11741, 12869, 13575, 12624, 8644, 9798, 10605, 14144, 15344, 14081, 14382, 12027, 12973\}$$
$$x_2 = \{11451, 10816, 11502, 11310, 10988, 11090, 12118, 9369, 10427, 9312, 10839, 11259, 11204, 11431, 11093, 10451, 11426, 11318\}$$

(a) Is there evidence that one of the valves has lower opening pressure than the other?

(b) Are the normality and homoscedasticity assumptions satisfied?

(c) Construct the 95% confidence intervals for the population means.

11. An experiment was performed to determine if a new technician was proficient in performing a critical test used to evaluate the effectiveness of vacuum cleaners. The test procedure is to uniformly distribute $100g$ of dirt consisting of fine stones, sand, and talc over two square yards of test carpet. The dirt is then rolled into the carpet with a 25 pound roller for three minutes. Finally, the carpet is vacuumed for three minutes using a prescribed motion and rate. The recovered dirt in grams is determined from the weight change of the filter bag before and after the vacuuming step. Test carpets are vacuumed aggressively and weighed between trials to guarantee that they are clean before the dirt is applied. To demonstrate the new (2) technician's proficiency his recovery was compared to that of an experienced (1) technician for eight different carpet samples. For the new technician to be considered proficient, the mean recovery difference between the technicians must be less than $4g$ with 90% confidence. The recovery data were:

	Plush		Multi-level		Shag		Level-loop	
Technician	1	2	1	2	1	2	1	2
1	55.3	54.4	58.2	65.0	23.5	25.7	76.3	70.5
2	56.1	55.6	61.2	63.3	23.6	27.7	74.9	69.6

(a) Is there evidence that the new technician is proficient?

(b) Construct and interpret the 90% confidence interval for the difference between the two technicians in the context of the $4g$ constraint.

12. The boxplot slippage tests for the two-sample location problem were presented without any considerable theoretical justification. Write a MINITAB exec macro that creates boxplots of two random standard normal samples of specified sample size with a specified difference in location.

    (a) Run the macro twenty times for samples of size $n = 5$ with differences between the population means of 0, 1, 2, and 3 standard deviations and record the number of times the boxes are overlapped, that is, the number of times that you have to accept $H_0 : \mu_1 = \mu_2$ or reserve judgment. Use these data to construct the approximate OC curve for the boxplot slippage test and interpret the OC curve.

    (b) Repeat Part a, but use samples of size $n = 40$ and the second boxplot slippage test.

13. Edit your macro from Problem 3.12 to display dotplots instead of boxplots and use the new macro to approximate the OC curves for Tukey's quick test for $n = 5$ and $n = 40$.

14. A MINITAB macro similar to the one created in Problem 3.12 was written to compare the performance of the two-sample t test, Tukey's quick test, and the two boxplot slippage tests. The macro considered 10,000 pairs of normal homoscedastic samples of size $n = 5$, 12, 30, and 80 with increasing differences between the two population means. The resulting approximate OC curves are shown in Figure 3.2. The OC curves labeled Box and Median correspond to the first and second boxplot slippage tests, respectively, and the horizontal scale (Δz) indicates the difference between the two population means in standard deviation units. That is, $\Delta z = 1$ corresponds to $|\mu_1 - \mu_2| = 1\sigma_x$.

    Interpret the OC curves with respect to their Type 1 and Type 2 error rates. Specifically, answer the questions:

    (a) What decision should be made when two boxes are completely slipped from each other, regardless of sample size?

    (b) What action should be taken if two boxes are not slipped from each other but a difference is still suspected?

    (c) Which two-sample test for location has the lowest Type 1 error rate?

    (d) Which two-sample test for location is the most sensitive to small differences between the population means?

    (e) Which two-sample test for location is most conservative, that is, has the least sensitivity to small differences between the population means?

    (f) What is the smallest sample size for which the second boxplot slippage test has tolerably low Type 1 error rate ($\alpha \le 0.05$)? What does this imply about the conditions under which the second boxplot slippage test can be safely used?

    (g) Why are the OC curves for the first boxplot slippage test and Tukey's quick test almost identical for $n = 5$?

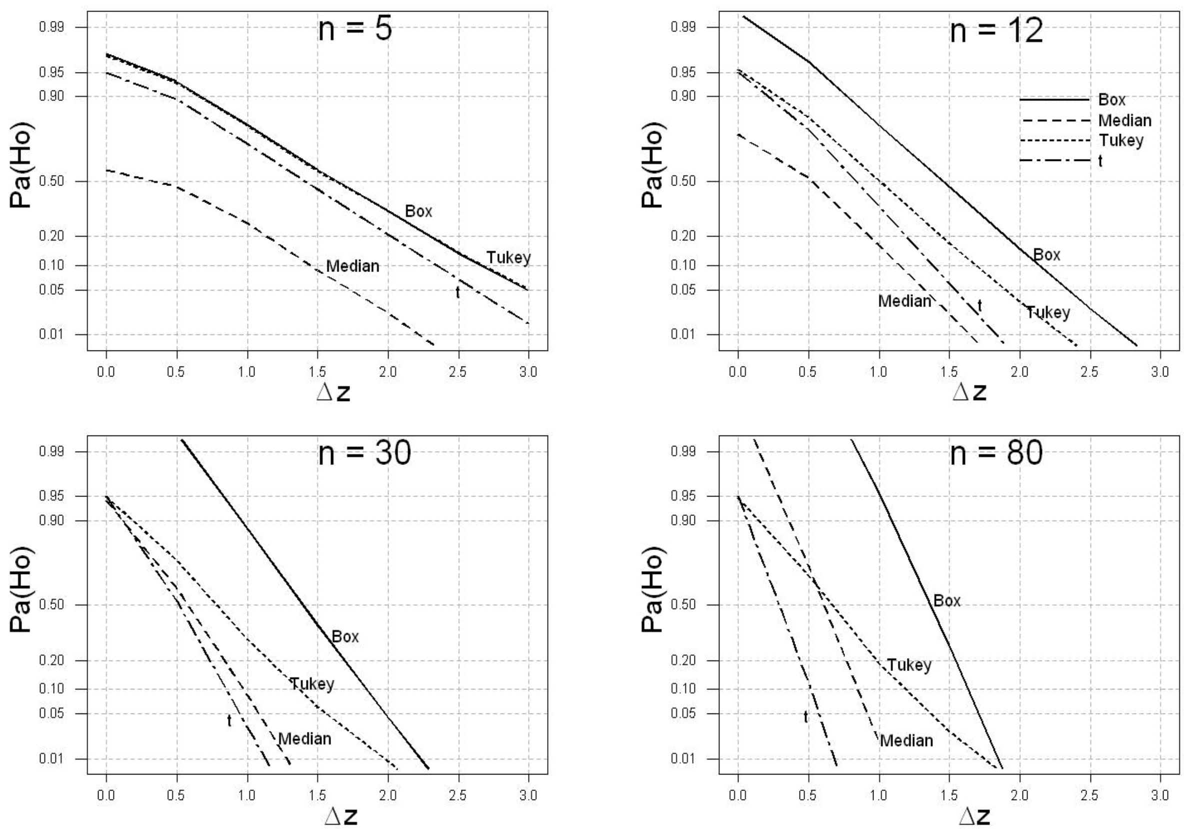

FIGURE 3.2. Operating characteristic curves for two-sample tests for location.

(h) When does the second boxplot slippage test have comparable performance in terms of Type 1 and 2 errors to the two-sample t test?

(i) How does the Type 1 error rate of Tukey's quick test change with sample size?

(j) How does the sensitivity of the different tests to small and large differences between the means change with increasing sample size?

15. A manufacturer wants to determine if material provided by two suppliers have different amounts of variation. He samples $n_1 = 12$ parts from the first supplier and $n_2 = 10$ from the second and finds the standard deviations to be $s_1 = 0.0045$ inches and $s_2 = 0.0081$ inches, respectively. Is there sufficient reason to believe that the second supplier's product is more variable than the first's?

16. Find the sample size required for a test for one sample mean where $\sigma_x = 50$ is known if a difference of $\delta = 12$ between the true and actual means must be detected with 90% probability. Use $\alpha = 0.05$ and a two-tailed test. Check your work with MINITAB.

17. Repeat Problem 3.16 with the power increased to 95% and $\alpha = 0.01$. What effect do the lower risks have on the sample size?

18. Use MINITAB to find the sample size for a test for one sample mean with unknown standard deviation if a difference of $\delta = 0.005$ between the true and actual means must be detected with 95% probability. Use a two-tailed test with $\alpha = 0.05$ and estimate the standard deviation with $\sigma_x \simeq 0.003$.

19. Use MINITAB to determine the sample size to detect a shift in a process mean from 1.300 inches to 1.320 inches or greater with 95% probability. Use $\alpha = 0.05$ and estimate the standard deviation as $\sigma_x \simeq 0.005$.

20. Find the sample size for a test for the difference between two population means ($\sigma_1 = \sigma_2$, but unknown) if a difference of $\Delta\mu = 400$ must be detected with 90% probability. Use $\alpha = 0.02$ and estimate the standard deviations with $\sigma \simeq 1200$.

21. Repeat Problem 3.20 if we must detect a difference of $\Delta\mu \geq 400$ with probability 95%.

22. It's very important not to reverse the numerator and denominator degrees of freedom when looking up values in tables for the F distribution because $F_{\alpha,\nu_1,\nu_2} \neq F_{\alpha,\nu_2,\nu_1}$, however, the F distribution does obey the property $F_{\alpha,\nu_1,\nu_2} = 1/F_{1-\alpha,\nu_2,\nu_1}$. For example, $F_{0.05,4,20} = 1/F_{0.95,20,4}$ where 0.05 and 0.95 indicate the areas in the same (left or right) tails of the F distribution. Use this property and Table A.5 from the book to find the F values for the following cases, where the subscript indicates the right tail area:

    (a) $F_{0.95,20,4}$

    (b) $F_{0.95,4,20}$

    (c) $F_{0.99,10,5}$

23. Student's t distribution is actually a special case of the F distribution. The t and F distributions are related by $t^2_{\alpha/2,\nu} = F_{\alpha,1,\nu}$ where $\alpha/2$ and α indicate the right tail areas of the t and F distributions, respectively. This trick has important applications in linear regression. Use this property to confirm the numerical equality of the following t and F values:

    (a) $t^2_{0.025,10} = F_{0.05,1,10}$

    (b) $t^2_{0.025,30} = F_{0.05,1,30}$

    (c) $t^2_{0.005,15} = F_{0.01,1,15}$

24. A common alternative to the two-sample t test for location is performed using of a pair of confidence intervals. If the intervals overlap, then $H_0 : \mu_1 = \mu_2$ is accepted or we reserve judgment, but if they are slipped, then H_0 is rejected. (This procedure has a similar flavor to the first boxplot slippage test.) What confidence level should be used to construct the confidence intervals if the conclusion drawn from the confidence intervals is to approximately match the conclusion from the t test?

25. Two-sample tests for location are a fundamental analysis tool of DOE. Create a comprehensive catalog of two-sample location tests and provide brief statements of the conditions required by each method. Be sure to add the nonparametric Mann-Whitney test to your list. Check MINITAB's **Stat**> **Nonparametric**> **Mann-Whitney**> **Help** menu for details on the Mann-Whitney test.

26. Match each test to its description:

Answer	Test
	paired-sample t test
	F test
	one-sample z test
	Boxplot slippage test
	χ^2 test
	two-sample z test
	Tukey's quick test
	two-sample t test
	Anderson-Darling test
	Satterthwaite's or Welch's test
	one-sample t test

(a) $H_0 : \mu_1 = \mu_2$ versus $H_A : \mu_1 \neq \mu_2$ when σ_1 and σ_2 are both known.

(b) A simple two-sample test for location performed with dot plots.

(c) Used to compare the mean of a population against a specified value when σ is known.

(d) Test to compare a population standard deviation to a specified value.

(e) A test for bias between two observers or methods.

(f) A two-sample location test that has different forms depending on the assumption of equal treatment variances.

(g) Based on the relative position of quartiles.

(h) A test for normality.

(i) A two-sample location test used when the populations are heteroscedastic.

(j) A test for homoscedasticity of two populations.

(k) The test statistic is given by $\frac{(\bar{x}-\mu_0)}{s/\sqrt{n}}$.

4

DOE Language and Concepts

1. Answer each of the questions regarding the experimental data shown in the table below. The observations were taken in the order that they appear in the table.

   (a) Was the experiment done in random or standard order?

   (b) How many replicates are there?

   (c) Were the runs blocked, and if so, were runs randomized differently between the different blocks?

   (d) Is the experiment balanced?

   (e) What pairs of variables are confounded?

   (f) Which variable is an uncontrolled covariate?

Order	Response	x1	x2	x3	x4	x5	Temperature
4	197	-1	1	1	1	-1	68
2	199	-1	-1	1	-1	-1	66
7	236	1	1	-1	1	1	74
6	226	1	-1	1	-1	-1	72
1	179	-1	-1	-1	-1	1	74
3	176	-1	1	-1	1	1	71
8	220	1	1	1	1	-1	74
5	196	1	-1	-1	-1	1	70
13	226	1	-1	-1	-1	1	71
15	216	1	1	-1	1	1	71
14	246	1	-1	1	-1	-1	66
10	182	-1	-1	1	-1	-1	71
16	237	1	1	1	1	-1	73
12	186	-1	1	1	1	-1	71
11	182	-1	1	-1	1	1	71
9	158	-1	-1	-1	-1	1	76

2. With respect to Example 4.9 in the textbook, design an improved experiment to determine the sensitivity of golf ball flight distance to temperature and explain how the two experiments differ.

3. In Problem 4.2, suppose that golf balls come in packages of six and that you are concerned that there might be significant differences between packages. How should this new factor be integrated into the experiment design?

4. An experiment with four unique runs in its design $(1, 2, 3, 4)$ is to be built. Each run is to be built three times. Write out possible orders for the experimental runs if:

   (a) The experimental runs are to be performed in completely random order.

   (b) The experiment is to be blocked on replicates.

   (c) The runs are to be built as repetitions.

5. A valve with proven field performance displayed a sudden increase in the number of units that failed a final leak test. The historical defective rate due to leaks was about 0.2% and the new defective rate was about 20%. The initial investigation into the problem showed that all methods and equipment had passed validation tests and that no changes to the process or leak measurement system had been made. Some components in the valves were known to have process capability problems and the process engineers felt that this was the likely cause of the leak problem, but production data showed that the severity of the known problems had not changed.

   To help identify the source of the problem, components for 100 units were randomly selected from production. Half of them were assembled in the production clean room and the other half were assembled in a laboratory clean room. Both clean rooms had been shown to deliver equivalent defective rates when the original assembly process was validated. When the 100 assembled valves were leak tested none of the fifty units assembled in the lab leaked and eight of the fifty units assembled in the production clean room leaked. After these results were reported an argument started over what the appropriate follow-up action should be. Some people wanted to measure critical dimensions on all 100 valves in an attempt to identify the variables or combinations of variables that caused the leaks. Other people felt that the production clean room assembly process should be studied. Which action is appropriate and what is the importance of the random assignment of valve components to the two treatment groups? Is there any benefit to pursuing both actions?

6. Identify a relatively simple problem regarding a location or variation difference between two treatments. Use the 11-step DOE process to study the situation and prepare a brief presentation documenting each step.

7. Match each DOE concept to its description:

Answer	**Concept**
	confounding
	interaction
	replicate
	variables matrix
	randomization
	nesting
	response surface
	repetition
	OVAT
	blocking
	screening
	design matrix

(a) Consecutive experimental runs.

(b) A type of experiment design used to find the few most important variables.

(c) When the effect of one variable depends on the level of another.

(d) Defines the design variable levels in coded units.

(e) An experimental method that can't resolve interactions.

(f) Two design variables that predict each other.

(g) The levels of one variable are unique within the levels of another.

(h) The run order used for study variables.

(i) A type of experiment design that can model curvature in the response.

(j) Observations taken under the same design variable settings but at different times.

(k) Relates the physical values of design variables to their coded values.

(l) A method of breaking up a large experiment into smaller sets of runs that are more likely to be made under homogeneous conditions.

5

Experiments for One-Way Classifications

1. Use the method of Section 5.4.1 to complete the following one-way ANOVA worksheet and calculate s_ϵ, r^2, and r^2_{adj}. Use a calculator to determine the required quantities. Check your work with MINITAB.

	Treatment			
Trial	A	B	C	D
1	87	43	70	67
2	70	75	66	85
3	92	56	50	70
$\sum y_i$	249	174		
$\bar{y}$	83	58		
s_i^2	133	259		

Source	df	$\widehat{\sigma}_y^2$	F	p
Treatment		$ns_{\bar{y}}^2 =$		
Error		$\overline{s_i^2} =$		
Total				

2. An experiment is designed to determine which of six different oils provides the best lubrication for a complex mechanism. Each oil is run in the mechanism eight times. The run order is completely random. Use this information to complete the following ANOVA table. Is there evidence that one or more of the oils is different from the others? (Use $\alpha = 0.05$.) Determine s_ϵ, r^2, and r^2_{adj}.

Source	df	SS	MS	F	p
Oil		4525			
Error		14742			
Total					

3. The concentration of active microorganisms in a suspension used in biological studies is determined by inoculating plates of growth media with samples from the suspension, incu-

bating the plates under the proper growth conditions for that organism, and then counting the number of colonies of microorganisms that grow.

An experiment was performed to compare four different methods of preparing tryptic soy broth (TSB) plates by making twelve plates using each method. All of the plates were inoculated with $1cc$ of the same spore suspension and incubated together. The 72 hour spore colony counts are shown in the table below. Is there evidence of a difference between the methods and if so, which methods are different from the others?

A	B	C	D
155	182	143	167
160	184	147	171
162	181	136	168
184	185	138	182
151	168	133	187
173	187	115	174
150	191	157	166
153	181	132	157
167	201	132	164
174	183	136	169
163	171	140	166
164	216	144	175

4. Four different plant fertilizers (A, B, C, D) were applied to randomly selected 1 acre slices of a 20 acre rectangular field planted with soybeans. Each fertilizer was used on 5 acres of field. The following table indicates the number of bushels harvested from each acre. Is there evidence that any of the fertilizers are different from the others? Be sure to check your assumptions and use a multiple comparisons test method to detect differences among the fertilizers if appropriate. If the 20 acre field is on a hillside how should the slices be oriented? Which fertilizers should you buy? Which fertilizers should you not buy?

A	B	C	D
32	26	32	33
33	28	36	30
31	26	38	33
34	22	33	36
31	33	35	31

5. A calculator manufacturer wants to evaluate four different calculator keyboard layouts to determine which design is the easiest to use. To measure ease of use they give a calculators to each of 32 engineers. The engineers are trained in the use of the calculator and agree to use the calculator exclusively for a period of three months. After the three month period the engineers are all given a timed test which they must use their calculator to complete. The problems are designed not to be hard, but to require extensive use of the calculator. The response is the amount of time that they take to complete the test. The results are shown

in the following table. Is there evidence that there are differences among the calculators?

A	43	49	59	51	47	54	54	51
B	47	53	52	47	46	52	52	51
C	55	56	53	46	58	49	52	50
D	36	38	41	42	41	39	42	38

6. The following table shows data taken in a completely randomized manner from 5 different processes. Analyze the data using a one way ANOVA and carefully check the assumptions about the equality of variances and the normality of the residuals. Are the assumptions met? If not, find a transformation that validates the assumptions and complete the ANOVA.

Row	*A*	*B*	*C*	*D*	*E*
1	67	90	25	55	30
2	37	244	30	60	74
3	40	110	27	74	81
4	27	164	67	45	60
5	40	121	55	45	49
6	55	67	74	37	67
7	33	81	33	45	25
8	45	90	40	55	33

7. An experiment was performed to compare the strength of quick connect glue bonds under four different surface preparation conditions: none, clean, prime/rough, and full. Six quick connects were prepared under each condition using the standard procedures associated with those conditions. After the assembled units had fully cured, they were pull-tested until failure. The pull test data are shown below. Is there any evidence of a difference between treatments? Recommend a follow-up experiment and calculate its sample size.

None	*Clean*	*P/R*	*Full*
72.1	81.2	99.4	82.2
61.1	86.3	100.7	87.7
72.6	74.4	94.5	102.2
80.1	89.8	84.4	96.0
75.3	85.7	84.6	91.0
83.7	81.8	90.3	94.4

8. Two students performed a science fair experiment to study the viscosity of household liquids (Source: John Swang, http://youth.net/nsrc/sci/sci043.html). The liquids that they considered were water, alcohol, oil, soap, and honey. To measure the viscosity, they filled a $21cm$ tall graduated cylinder with one of the experimental liquids and measured the amount of time in seconds it took a $5.7g$ $1.5cm$ diameter marble to fall to the bottom. Use ANOVA to analyze the data. Be careful to validate the ANOVA method by inspecting the distribution of the residuals. Is there sufficient evidence to conclude that the viscosities of

water and alcohol are different? Oil and soap?

Trial	*Water*	*Alcohol*	*Oil*	*Soap*	*Honey*
1	0.89	0.55	4.04	3.54	71
2	0.61	0.51	3.72	3.33	89
3	0.72	0.62	3.68	4.81	73

9. A one-way ANOVA is planned to compare five different treatment groups for a possible difference between their means. The standard deviation of the inherent noise is known to be $\sigma_\epsilon = 30$. How many units from each treatment must be run in order to have a 90% chance of detecting a difference of $\delta = 20$ between a pair of treatments? Use the sample size calculation method described in Chapter 5 and compare your result to that from MINITAB.

10. A one-way classification experiment to be analyzed by ANOVA can only have $n = 14$ replicates in each of its $k = 4$ treatments. Use MINITAB's **Stat> Power and Sample Size> One-Way ANOVA** menu to create the operating characteristic curve $Power\,(\delta)$ for this experiment if $\sigma_\epsilon = 20$. How large a difference between a pair of means is required so that the experiment has 90% power?

11. Figure 3.2 indicates that the first boxplot slippage test gives excellent protection against Type 1 errors for samples of size $n > 5$. What does this observation imply about the safely of post-ANOVA multiple comparisons using boxplot slippage tests? What is the disadvantage of using only this method?

12. The preferred method of analysis to test for a location difference between two treatments is the two-sample t test and the preferred method for three or more treatments is ANOVA, however, the two-sample t test and ANOVA are equivalent to each other in the case of two treatments. The relationship between the test statistics from the two methods is:

$$F_{\alpha,1,df_\epsilon} = t^2_{\alpha/2,df_\epsilon}$$

Use this relationship to compare the results of the two-sample t test and ANOVA analyses of Problem 3.9. Be sure to compare the p values of the tests, too.

13. Match each post-ANOVA multiple comparisons method to its description:

Answer	**Method**
	Duncan's method
	Tukey's HSD test
	Bonferroni's method
	Hsu's test
	two-sample t test
	Sidak's method
	Dunnet's test
	Fisher's method

(a) Compares $k - 1$ of the k treatments to the best treatment.

(b) Reduces the overall Type 1 error rate by the number of comparisons.

(c) Most sensitive of the methods for $\binom{k}{2}$ comparisons.

(d) The user specifies one Type 1 error rate for each of the $\binom{k}{2}$ tests.

(e) Safe, but not as conservative as another method that it is often confused with.

(f) Not considered safe unless the Type 1 error rate is adjusted.

(g) Compares $k-1$of the k treatments to a specified control treatment.

(h) Compares all $\binom{k}{2}$ pairs of treatment means, most sensitive than the most conservative method but less sensitive than another.

14. Example 5.12 in the textbook suggests two transforms for Poisson-distributed count data that recover the homoscedasticity of the response with respect to the response magnitude.

    (a) Use MINITAB to simulate Poisson-distributed count responses over a wide range of values and determine estimates for the standard deviations of the transformed responses.

    (b) Use the results from Part a) to determine the sample size required to test $H_0 : \lambda_1 = \lambda_2$ versus $H_A : \lambda_1 < \lambda_2$ if the experiment must have 90% power to distinguish $\lambda_1 = 4$ from $\lambda_2 = 9$.

    (c) Write a general equation to determine the sample-size for the two-sample count response problem.

6

Experiments for Multi-Way Classifications

1. An experiment was performed to study the storage effects of humidity and temperature on the degradation of photographic film. The experiment considered three levels of humidity and four levels of temperature. Three measurements of color rendition were made at each humidity and temperature combination. The ANOVA sums of squares are shown in the table below. Complete the ANOVA table and interpret it. What issues might compromise the validity of this experiment? If you had to prepare an analysis and write an official report on this experiment based on the information given in the problem statement, what disclaimers would you include?

Source	df	SS	MS	F	p
Humidity		12200			
Temperature		45000			
Interaction		3300			
Error		24000			
Total					

2. The strength of a special fabric was measured after being washed in water and detergent at different temperatures and pH. The ANOVA output from the experiment is shown below. Complete the ANOVA table and calculate the standard error, coefficient of determination, and adjusted coefficient of determination. Interpret the ANOVA if:

   (a) Temperatures were run in random order and pH was run as a blocking variable.

   (b) pH levels were run in random order and temperature was run as a blocking variable.

   (c) Both variables were blocking variables.

   (d) The levels of both temperature and pH were run in completely random order.

Source	df	SS	MS	F	p
Temperature	2	8000			
pH	2	680			
Interaction		500			
Error		1800			
Total	26				

3. An injection molding machine has five cavities that are supposed to be identical to each other. Each cycle of the machine produces one part from each cavity. In order to check that the five cavities are producing parts with the same dimension, parts are pulled from all five of the cavities and measured. Samples are taken from a total of eight cycles of the machine. The data are shown below. Is there evidence of differences between the cavities? Is there evidence of an interaction between the cavity and the cycle?

$Cycle$	$Cavity1$	$Cavity2$	$Cavity3$	$Cavity4$	$Cavity5$
1	99	104	113	125	122
2	119	113	123	143	134
3	150	122	137	101	134
4	102	119	134	136	117
5	115	126	113	153	122
6	131	89	114	136	141
7	112	113	136	146	120
8	139	133	90	125	96

4. A sailboat manufacturer wishes to identify a single epoxy that has high strength at all temperatures. He designs a factorial experiment to evaluate the strength of epoxies from three different manufacturers at low, intermediate, and high temperatures. He performs the experiment in a completely randomized manner by randomly selecting an epoxy and temperature for each run until all experimental runs are completed. The strength data are shown below. Analyze the data by two-way ANOVA and construct an interaction plot. Which manufacturer should he use and what considerations should be made?

		Manufacturer		
		A	B	C
Temperature	20	216, 239, 234	278, 299, 271	309, 295, 315
	25	344, 335, 348	311, 319, 327	321, 312, 325
	30	372, 366, 385	360, 366, 361	371, 361, 349

5. Suppose that the experiment in Problem 6.4 had been blocked on replicates, where the three observations under each experimental condition were collected in the order shown. Is there evidence of a block effect and does it change the original analysis and interpretation?

6. Reanalyze the data from Problem 3.11 as a 2×8 factorial design by ignoring the different carpet types. How do the results of this analysis compare to the results from the paired sample t test analysis? Do you learn anything more from this analysis that you didn't learn from the paired sample t test analysis?

7. (This problem was moved to Chapter 7.) Reanalyze the data from Problem 3.11 as a 2×4 factorial design (technician vs. carpet type) with carpet samples nested within carpet type. How does this analysis compare to that of the analysis as a 2x8 factorial design?

8. An experiment was performed to compare the lumen measurements obtained by four photometry labs. A collection of six lamps was prepared and circulated to each of the labs. Each lab measured each lamp twice and measurements were made in completely random order. Analyze the data and interpret the results. Construct an appropriate error statement for this data set.

		Lab			
		A	B	C	D
Lamp	#71522	2409, 2494	2465, 2693	2499, 2365	2556, 2498
	#71533	4477, 4182	4485, 4283	4131, 4076	4297, 4481
	#71534	8861, 8739	9638, 9084	9272, 8904	9579, 8479
	#71535	10213, 10281	11138, 11560	10479, 10468	11151, 11015
	#71536	20601, 22996	23797, 23625	22106, 20773	20884, 22430
	#71537	35985, 40224	42457, 41064	39140, 37987	39049, 37204

9. Water samples were drawn from three wells at an industrial site for the purpose of determining water contamination levels. Initial water samples were drawn from each well, then the wells were pumped for twenty minutes and second samples were taken. Each sample was assayed for three contaminants. The chemical assays are expensive so there is interest in reducing the number of measurements that have to be made in future evaluations at the site. The contaminant concentrations in ppm are shown in the table below. Analyze the data and interpret the results.

		Trial	
Site	Contaminant	1	2
A	BZME	860	2600
A	FC113	990	5600
A	TCE	23	120
B	BZME	100000	90000
B	FC113	19000	18000
B	TCE	4600	3900
C	BZME	37	98
C	FC113	2	63
C	TCE	10	10

10. A custom macro called *power.mac* is included in the *MINITAB 14 Macros* folder on the CD ROM provided with this book. The macro performs power and sample size calculations for balanced fixed effects ANOVA problems. Open the macro in Notepad to view the instructions for running it. Then use the macro to solve the following problems:

    (a) Determine the power provided by three replicates of a 2×5 balanced full factorial design to detect a difference of $\delta = 10$ between the levels of the first (two-level) treatment if $\sigma_\epsilon = 12$.

(b) For the situation describe in Part a, how many replicates are required to achieve 90% power?

(c) Determine the power to detect a difference of $\delta = 10$ between two levels of the second (five-level) variable in Part a.

(d) For the situation describe in Part c, how many replicates are required to achieve 90% power?

7
Advanced ANOVA Topics

1. Complete the ANOVA table for each situation:

   (a) A and B are both fixed variables.

$Source$	df	SS	MS	F	p
A	3	372			
B	2	84			
$A \times B$		96			
$Error$		1080			
$Total$	119				

   (b) A is fixed and B is random.

$Source$	df	SS	MS	F	p
A	3	372			
B	2	84			
$A \times B$		96			
$Error$		1080			
$Total$	119				

   (c) A and B are both random.

$Source$	df	SS	MS	F	p
A	3	372			
B	2	84			
$A \times B$		96			
$Error$		1080			
$Total$	119				

2. A Latin square experiment was performed using four levels of each of the three experimental variables. The data are shown in the table below. Variable A is the study variable and variables B and C are blocking variables. Is there sufficient evidence to indicate that there are differences between any of the levels of A?

A	B	C	Y
1	1	1	616
1	2	2	554
2	4	1	681
3	3	1	638
3	4	2	590
2	2	3	555
2	3	4	462
2	1	2	586
1	3	3	532
3	2	4	444
1	4	4	524
3	1	3	446
4	1	4	471
4	3	2	579
4	2	1	653
4	4	3	567

3. A gage error study was performed using five parts, three operators, and two trials. Analyze the GR&R data, which are shown below. Use a process tolerance of 2.0 measurement units.

Part	*Trial*	*Op*1	*Op*2	*Op*3
1	1	10.27	10.26	10.15
1	2	10.40	10.40	10.27
2	1	9.88	9.91	9.95
2	2	10.07	9.94	9.94
3	1	10.26	10.27	10.10
3	2	10.28	10.26	10.21
4	1	10.13	10.21	10.16
4	2	10.15	10.33	10.18
5	1	9.35	9.29	9.31
5	2	9.40	9.40	9.30

4. The macro *grrsim.mac*, which is in the *MINITAB 14 Macros* folder on the CD ROM distributed with this book, performs a simulation of a GR&R study using the specified number of parts, operators, and trials. The calling statement for the macro for ten parts, three operators, and two trials is:

```
mtb> %grrsim 10 3 2 c1-c3
```

where the output from the macro goes in columns `c1-c3`. Run the simulation for this condition and analyze the data using **Stat> ANOVA> Balanced ANOVA**, **Stat>

ANOVA> General Linear Model, and **Stat> Quality Tools> Gage Study> Gage R&R Study (Crossed)**. Set the process tolerance to be 200. Be sure to declare part and operator as random variables in the ANOVA menus and to turn on the variance components reports in the **Results** menus. Refine the models by eliminating any insignificant terms and then confirm that all three methods of analysis are equivalent. Start from the variance components report from either ANOVA output and confirm the calculations of %Contribution, StdDev, Study Var, %Study Var, and %Tolerance in the GR&R study output.

5. A gage error study was performed to compare the measurements made by three different contract chemistry labs. It was impractical to transport the same samples to all of the labs, so ten random samples from the same homogeneous lot were sent to each lab. Analyze the nested factorial experiment in MINITAB using each of the following methods and confirm that they give the same answers. Assume that the process tolerance is 5.0 units. The data are in the table below.

   (a) **Stat> ANOVA> Fully Nested ANOVA**.

   (b) **Stat> ANOVA> General Linear Model**.

   (c) **Stat> Quality Tools> Gage Study> Gage R&R Study (Nested)**.

Part	*Trial*	*Lab*1	*Lab*2	*Lab*3
1	1	38.7	40.8	37.0
1	2	38.6	40.8	36.9
2	1	37.6	37.5	37.8
2	2	37.6	37.5	37.5
3	1	37.3	36.2	37.9
3	2	37.4	36.0	37.9
4	1	36.4	39.0	37.5
4	2	36.7	39.2	37.4
5	1	38.2	39.0	38.1
5	2	38.0	38.9	38.0
6	1	38.0	39.3	37.5
6	2	38.1	39.2	37.6
7	1	36.9	37.1	39.4
7	2	36.8	37.2	39.3
8	1	36.6	38.9	38.2
8	2	36.7	39.0	38.0
9	1	38.0	37.4	37.3
9	2	38.2	37.4	37.5
10	1	37.3	38.3	38.4
10	2	37.3	38.3	38.4

6. Although the most common method for interpreting the gage error (GRR) after a gage error study is to compare it to the tolerance, other interpretation methods are occasionally used . In particular, if the part variation is relatively small compared to the tolerance, then PV provides a more appropriate basis of comparison for GRR. A special statistic called the *number of distinct categories* or NDC is used to estimate the number of categories that

parts with variation indicated by the PV value could be sorted into. Measurement systems with $NDC \geq 5$ are considered to be acceptable. NDC is calculated from:

$$NDC = \sqrt{2} \times trunc\left(\frac{PV}{GRR}\right)$$

where the $trunc\,()$ function rounds its argument down to the nearest integer. Use this calculation to confirm the NDC value reported by MINITAB in Example 7.6 in the textbook.

7. The usual maximum allowed for repeatability and reproducibility in gage error studies is 10% of the tolerance. Suppose that a process has repeatability equal to 10% of the tolerance and reproducibility equal to 20% of the tolerance. Find the power of the ANOVA F statistics to detect these rejectable conditions for the following gage error study designs and use your findings to write guidelines for GR&R study designs.

   (a) Ten parts, two operators, three trials.

   (b) Ten parts, three operators, three trials.

   (c) Ten parts, three operators, two trials.

   (d) Six parts, five operators, two trials.

8. Reanalyze the data from Problem 3.11 as a 2×4 factorial design (technician vs. carpet type) with carpet samples nested within carpet type. How does this analysis compare to that of the analysis as a 2x8 factorial design (Problem 6.6)?

8
Linear Regression

1. The light output in lumens from an arc lamp is a function of the input power in watts. Fit a linear model to the following data:

$Power$	60	40	70	50	30.3	23.2
$Lumens$	5324	2746	6441	4054	1565	937

The input power to these lamps is dissipated as radiation in the UV, Visible, and IR spectrum and as heat conducted to the arc lamp walls. If the ratios of UV, Visible, and IR power are constant (a good assumption) and if conducted power is independent of input power (another good assumption) use your model to estimate the power loss by conduction. Use your model to predict the light output at $55W$ and construct the 95% prediction interval at this wattage.

2. An experiment was performed to determine how the cutting speed of a tool v in feet per minute affected the lifetime of the tool t in minutes. The operation was run at 40 to 110 feet per minute in 10 foot per minute increments and tools life was recorded to the nearest minute. Three observations were taken at each speed and the order of the observations was completely randomized. The data are shown in the following table. The "*" indicates that the tool was broken before it wore out. Analyze the data to determine how tool life depends on speed. What speed gives the longest life? Why wouldn't you run at that speed all of the time? Use a quadratic model and the linear goodness of fit test to check for lack of fit.

$Speed(ft/min)$	40	50	60	70	80	90	100	110
$ToolLife(min)$	161	144	136	137	121	114	108	13*
	153	146	138	139	118	119	105	75*
	152	148	137	128	123	115	104	21*

3. The mechanical properties of a material are determined from its stress-strain curve (y vs. x) where stress τ is the load in pounds W divided by the cross sectional area A of the rod:

$$\tau = \frac{W}{A}$$

and the strain (ϵ) is the relative elongation:

$$\epsilon = \frac{\Delta l}{l_0}$$

where l_0 is the initial length of the rod and Δl is the amount of elongation. A typical plot of stress versus strain (τ vs. ϵ) shows a linear relationship between stress and strain for small applied stresses followed by a nonlinear region for large stresses. The critical stress that separates the linear and nonlinear regions is called the yield stress. Applied stresses less than the yield stress do not cause permanent deformation of the material. Applied stresses greater than the yield stress cause permanent deformation of the material. The slope of the stress-strain curve in the linear region is called the Young's modulus or the modulus of elasticity (Y):

$$Y = \frac{\Delta \tau}{\Delta \epsilon}$$

In order to determine the tensile strength properties of brass, a $2in$ long brass rod $0.505in$ in diameter is loaded along its length. As the size of the load is increased the elongation of the rod is measured. The load in pounds and the elongation in inches are shown in the following table (Doyle, Manufacturing Process and Materials for Engineers, Prentice-Hall, 1969). Find the Young's modulus and yield stress of the material. Construct and interpret two models - one with and one without the constant.

W (lbs)	Δl (in)
320	0.0002
590	0.0004
920	0.0006
1310	0.0008
1600	0.0010
1880	0.0012
2270	0.0014

W (lbs)	Δl (in)
2560	0.0016
2760	0.0018
2920	0.0020
3020	0.0030
3080	0.0040
3220	0.0060
7220	0.3000

4. The following table shows the waterline length LWL and the measured maximum speed of displacement (i.e. non-planing) boats (McGraw-Hill Encyclopedia of Science and Technology, 1960). Plot the speed versus the waterline length and note that the relationship is not linear. Fluid mechanics suggests that the speed should be proportional to the square root of the waterline length. Transform the waterline length and plot the data to confirm this model is appropriate. Fit a regression model to the data and use your model to predict the maximum speed of a $36ft$ boat. Construct a 95% confidence interval for the speed of the boat.

$LWL(feet)$	22	30	34	48	65	105	130	240	410	980
$Speed(knots)$	6.1	6.6	6.3	8.1	8.9	13.4	15.4	20	28	42

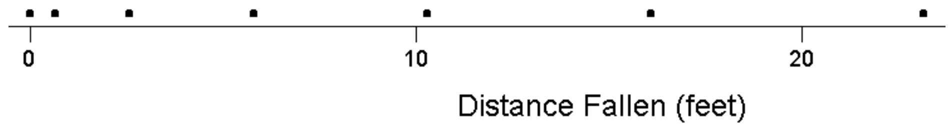

FIGURE 8.1. Stroboscopic Position of Falling Ball in $\Delta t = 0.2s$ Intervals

5. Figure 8.1 shows the position of a dropped ball in feet as imaged by a stroboscope with a $0.2s$ strobe interval. Extract data from the figure and plot the distance fallen h versus the total elapsed time t. Note that the relationship is not linear. Physics predicts that the relationship between h and t is $h = \frac{1}{2}gt^2$ where $g = 32.15ft/s^2$ is the acceleration of gravity at the surface of the earth.

   (a) Transform the time by squaring it and plot h versus t^2. Fit a model for h as a function of t^2. (Fit a model of the form $h = bt'$ where $t' = t^2$.) Suppress the constant term by selecting the **Remove Intercept** option in the **Analysis> Regression/Correlation> Linear Regression** menu. Use your model to estimate the acceleration of gravity g and construct a 95% confidence interval for it.

   (b) Fit a model of the form $h' = a + bt'$ where $h' = \log(h)$ and $t' = \log(t)$. Use your model to estimate the acceleration of gravity and the exponent of time. Determine the 95% confidence interval for the exponent of time and compare it to its expected value from gravitational theory.

6. In order to determine the effectiveness of a chemical sterilant, biological organisms were added to a solution of the sterilant. At two minute time intervals a $1cc$ sample of the organism/sterilant solution was drawn and neutralized so that the sterilant wouldn't kill the organisms any more. Then the solution was transferred to a petri dish of growth medium where the surviving organisms were cultured for three days and finally counted. The whole process was repeated three times. From the data in the following table: a) Estimate the mean number of organisms per cc at $t = 0$. b) Estimate the number of organisms remaining at 3 minutes and construct the 95% confidence interval. c) Estimate the slope of the line and construct the 95% confidence interval. d) At what time should all of the organisms be killed? (Hint: Use the model $N(t) = N_0 10^{-t/\tau}$.)

$Time(min)$	0	2	4	6	8	10	12
$Number$	460000	25000	3700	260	55	4	0
of	490000	82000	920	370	17	0	0
$Survivors$	130000	96000	6100	450	26	3	0

7. A 5th grade student performed a science fair experiment to study how high a dropped basketball rebounded as a function of the air pressure in the ball (Source: John Swang, http://youth.net/nsrc/sci/sci042.html). The ball was filled to pressures from 0 to $10psi$ and the ball was dropped from a constant height of three feet. Three drops were performed at each pressure and the rebound height in centimeters was determined from a videotape record of the bounces. The data are shown in the table below. Construct a model for the rebound height as a function of fill pressure. If you cannot find an appropriate variable transformation that linearizes the problem, try using the macro *fitfinder.mac* which is on

the CD ROM in the *MINITAB 14 Macros* folder. What evidence is there that the three trials were run as repetitions instead of as replicates?

	Trial		
Pressure(psi)	1	2	3
0	39.4	40.0	39.4
1	53.3	54.6	54.6
2	57.8	58.4	59.1
3	67.3	68.0	67.3
4	71.1	71.1	71.1
5	72.4	73.0	71.1
6	78.7	78.7	78.7
7	80.0	80.7	80.7
8	81.3	81.3	81.9
9	83.8	83.8	83.8
10	83.8	85.1	86.4

8. The average velocity v of a sailboat around a closed race course is related to the effective wind speed EWS by:

$$\frac{1}{v} = a + \frac{b}{EWS^c}$$

where a, b, and c are regression coefficients. Experimental data showing only the results from a boat's twenty best races are shown in the table below. This expression cannot be linearized, so the usual method of fitting the data must be modified. Assume a value for c (values between 1.4 and 2.2 are typical), transform the resulting equation to linear form, and fit the data to determine a and b. Repeat these steps for several choices of c and determine the value of c that minimizes SS_ϵ. This algorithm demonstrates manually what software that can do nonlinear fits does automatically. MINITAB can do limited nonlinear fits but it cannot solve this particular problem.

Race	*EWS*	*v*
1	11.1	5.44
2	15.1	5.94
3	10.2	5.40
4	10.6	5.43
5	15.9	6.07
6	16.4	6.12
7	2.0	2.28
8	25.3	4.94
9	3.2	5.90
10	8.4	5.44

Race	*EWS*	*v*
11	14.0	5.90
12	10.3	5.44
13	8.3	4.98
14	4.8	3.48
15	8.4	4.99
16	14.0	5.76
17	4.5	3.28
18	10.8	5.57
19	8.1	4.95
20	7.0	4.52

9. Preliminary data indicate that a response y depends on a variable x as $y \simeq 2100 + 35x$ where the range of x of interest is $10 \le x \le 20$. The standard error is estimated to be $\sigma_\epsilon \simeq 30$. The sample size must be large enough so that the 95% confidence interval for the slope β_1 is no wider than $\pm 0.05\beta_1$.

(a) Find the minimum sample size using an equal number of observations at just two extreme levels of x.

(b) How many observations are required if they are taken uniformly over the allowed range of x?

(c) How many observations are required for three evenly spaced levels of x?

(d) Compare the sample size obtained for a) to the sample size if x was limited to the interval $12.5 \leq x \leq 17.5$.

10. A standard incandescent light bulb was operated at several powers values (in watts) and its light output (in lumens) and its color temperature (closely related to the actual tungsten filament temperature, in degrees Kelvin) were measured. The data are shown below.

(a) Fit a regression model for the lumens as a function of power (P).

(b) The theoretical relationship between the power and the filament temperature (T) is given by $P = \epsilon\sigma AT^4$ where ϵ is the tungsten filament emissivity, σ is the Stefan-Boltzmann constant, and A is the filament surface area. Use the theoretical relationship to fit a model for the power as a function of temperature.

P	*Lumens*	T
82.9	592	2461
155.3	2441	2849
117.1	1322	2668
9.7	1	1524
28.2	32	1920
163.4	2718	2883
53.0	191	2219

11. (This problem is outside the scope of the textbook, but it provides an easy and striking example of analysis by binary logistic regression.) When an experimental response has only two states, often called a binary or dichotomous response, the usual regression and ANOVA methods of analysis are inappropriate because the distribution of residuals is non-normal. An alternative method of analysis for this situation, called *binary logistic regression* (BLR), uses a transformation of the binary response data to obtain response success probabilities (p) which *can* be analyzed correctly by modified regression and ANOVA methods. For a single predictor variable x the form of the BLR model is:

$$\ln\left(\frac{p_i}{1-p_i}\right) = b_0 + b_1 x_i + \epsilon_i$$

where $\ln(p/(1-p))$ is called the *logistic* or *log-odds* transform.

On 28 January 1986 the space shuttle Challenger exploded during take-off. The night before the accident NASA flight engineers tried to stop the launch because of the potential for an o-ring seal failure due to the low ambient temperatures prior to launch time. Management over-ruled the flight engineers and decided to launch anyway.

The table below shows the number of o-ring seal failures (f) out of the $n = 6$ o-ring seals on each flight prior to the Challenger accident with the corresponding temperature at the time of the launch. Use MINITAB's **Stat>** **Regression>** **Binary Logistic Regression** menu to build a binary regression model for the o-ring seal failure probability. Use your model to predict the failure probability at $26°F$ - the temperature at the time of the launch. Was there sufficient evidence to postpone the launch?

$Flight$	T	n	f
1	66	6	0
2	70	6	1
3	69	6	0
4	80	6	0
5	68	6	0
6	67	6	0
7	72	6	0
8	73	6	0
9	70	6	0
10	57	6	1
11	63	6	1
12	78	6	0

$Flight$	T	n	f
13	70	6	1
14	67	6	0
15	53	6	2
16	75	6	0
17	67	6	0
18	70	6	0
19	81	6	0
20	76	6	0
21	79	6	0
22	75	6	0
23	76	6	0
24	58	6	1

9

Two-Level Factorial Experiments

1. An experiment was performed to determine how the cure time of a two part epoxy depends on the resin to hardener ratio (R/H) and the temperature. The experiment was performed under controlled conditions and the runs were performed in completely randomized order. The cure time was taken to be the time required for the epoxy to harden to a specified Rockwell hardness. Analyze the following cure time data (minutes) and use your model to predict the cure time and its 95% prediction interval at 21C and 4.5:1 R/H.

		R/H	
		3:1	5:1
Temperature (C)	20	230, 210, 240	170, 180, 190
	25	180, 150, 170	140, 150, 150

2. A 2^3 experiment with two replicates in blocks was performed to study the flight time of the paper helicopter in helicopter2.doc. The design variables and their levels and shown in the following table. The width and length variables refer to the blade geometry and the folds variable indicates the number of one inch folds in the helicopter's drop leg.

$Variable$	-1	$+1$	$Units$
$A: Width$	1.25	2	$inch$
$B: Length$	2	4	$inch$
$C: Folds$	1	2	NA

The experimental flight times were measured in seconds and are reported below. Analyze the data and include a term for blocks in the model. What helicopter geometry is predicted to give maximum flight time? Extrapolate your model to recommend another helicopter geometry that would give even longer flight time. What are the risks associated with this recommendation?

			Block	
A	B	C	1	2
$-$	$-$	$-$	2.99	2.90
$-$	$-$	$+$	2.99	2.99
$-$	$+$	$-$	5.13	5.58
$-$	$+$	$+$	5.21	5.41
$+$	$-$	$-$	2.82	2.93
$+$	$-$	$+$	2.95	2.62
$+$	$+$	$-$	4.41	4.62
$+$	$+$	$+$	4.39	4.97

3. An experiment was performed (Said Jahanmir, NIST Ceramics Division: Material Science and Engineering Laboratory, www.itl.nist.gov/div898/handbook/pri/section4/pri471.htm) to study the effect of grinding on the strength of a high performance silicon nitride ceramic material. The purpose of the study was to determine the best settings of five grinding variables to maximize the strength of the material and to develop a model that expresses the strength as a function of those variables. The grinding variables considered in the study and their levels are shown below. A single replicate of a 2^5 full factorial experiment design was used and the experimental trials were randomized over all 32 runs. The experimental data are shown in Table 9.1. Analyze the data and refine the model. Make a recommendation on what variable levels should be used to maximize the material strength.

$Variable$	-1	$+1$	$Units$	Description
$x_1 : Batch$	A	B	NA	two material batches
$x_2 : Direction$	$long.$	$trans.$	NA	grinding direction
$x_3 : Grit$	$140 - 170$	$80 - 100$	NA	grinding material coarseness
$x_4 : Feed$	0.05	0.125	mm/min	material sample feed rate
$x_5 : Speed$	0.025	0.125	m/s	grinding table speed

Std	Run	x_1	x_2	x_3	x_4	x_5	$Y : Strength$
1	17	-	-	-	-	-	680
2	30	-	-	-	-	+	722
3	14	-	-	-	+	-	702
4	8	-	-	-	+	+	667
5	32	-	-	+	-	-	704
6	20	-	-	+	-	+	642
7	26	-	-	+	+	-	693
8	24	-	-	+	+	+	669
9	10	-	+	-	-	-	492
10	16	-	+	-	-	+	476
11	27	-	+	-	+	-	479
12	18	-	+	-	+	+	568
13	3	-	+	+	-	-	445
14	19	-	+	+	-	+	410
15	31	-	+	+	+	-	429
16	15	-	+	+	+	+	491
17	12	+	-	-	-	-	607
18	1	+	-	-	-	+	621
19	4	+	-	-	+	-	611
20	23	+	-	-	+	+	638
21	2	+	-	+	-	-	585
22	28	+	-	+	-	+	586
23	11	+	-	+	+	-	602
24	9	+	-	+	+	+	608
25	25	+	+	-	-	-	443
26	21	+	+	-	-	+	434
27	6	+	+	-	+	-	418
28	7	+	+	-	+	+	511
29	5	+	+	+	-	-	392
30	13	+	+	+	-	+	343
31	22	+	+	+	+	-	386
32	29	+	+	+	+	+	447

TABLE 9.1. NIST Ceramic Strength Experiment

4. Complete the following steps in the analysis of a three-variable design situation:

   (a) Determine the number of replicates required to detect a difference of $\delta = 10$ between the levels of main effects with 90% power in a 2^3 full factorial design when $\sigma_\epsilon = 10$.

   (b) Design the experiment determined in Part a in the Excel design file *2^3f.xls* and run the *sim3* simulation. Copy the design and response to MINITAB and analyze the data. Refine the model to include only the statistically significant effects ($p \leq 0.05$).

   (c) Use the model determined in Part b to predict the response when $(x_1, x_2, x_3) = (-1, -1, +1)$ and compare the prediction to the observations from this cell of the experiment. How well do they agree?

   (d) Use your model determined in Part b to predict the response when $(x_1, x_2, x_3) = (0, 0, 0)$. Run the *sim3* macro again with eight runs of $(x_1, x_2, x_3) = (0, 0, 0)$ and compare the predicted and observed values. How well do they agree and why?

5. Run the *sim3* macro for two replicates of a 2^3 design with the standard error set to $\sigma_\epsilon = 1$. Analyze the data and then rerun the macro and analysis with $\sigma_\epsilon = 4$, 16, and 64. Compare the ability of the different experiments to resolve the regression coefficients.

6. Run the *sim3* macro for one replicate of a 2^3 design with the standard error set to $\sigma_\epsilon = 10$ and analyze the data. Rerun the macro using 2, 4, and 8 replicates of the 2^3 design and compare the ability of the different experiments to resolve the regression coefficients.

7. Run the *sim5* macro for one and two replicates of the 2^5 design. Be sure that the standard deviation is set to 10. Analyze the data for main effects and two-factor interactions and refine the models to eliminate statistically insignificant terms. Transcribe the final regression coefficients and their summary statistics into the columns *2^5fr1* and *2^5fr2*, respectively, of the worksheet in *Homework Problems> sim5 Worksheet.xls*. Use the codes suggested on the worksheet to indicate the statistical significance of each regression coefficient.

8. Run the *sim5* macro for one replicate of a 2^5 design (use *2^5f.xls*) with the standard error set to $\sigma_\epsilon = 1$. Analyze the data and save the regression coefficients in the MINITAB worksheet, then refine the model to eliminate the insignificant terms. Rerun the macro and analysis with $\sigma_\epsilon = 3$, 10, and 30 and compare the performance of the different experiments. Construct and compare the normal probability plots of the regression coefficients from the full models excluding the model constant. What is the effect of increased standard error on the ability of an experiment to resolve model coefficients?

9. Run the *sim5* macro for 1, 2, 4, and 8 replicates of a 2^5 design (use *2^5f.xls*) with the standard error set to $\sigma_\epsilon = 10$. Analyze the data from each experiment, save the regression coefficients in the MINITAB worksheet, and then refine the model to eliminate the insignificant terms. Compare the abilities of the different experiments. to resolve the regression coefficients. Construct and compare the normal probability plots of the regression coefficients from the full models excluding the model constant. What is the effect of increased standard error on the ability of an experiment to resolve model coefficients?

10. Occam's razor provides a mandate to simply models, but when models have many variables and interactions it can be difficult to decide whether a variable and all of its associated interactions deserve to be kept or not. A simple lack of fit test to determine if a variable

and all of its interactions may be completely dropped from a model test the hypotheses H_0 : *the variable does not contribute significantly to the model* vs. H_A : *the variable does contribute significantly to the model.* The test procedure is:

1. Fit the full model (indicated with the letter F) including all terms involving the variable in question. The error sum of squares for this model is $SS_{\epsilon(F)}$ and the error degrees of freedom are $df_{\epsilon(F)}$.
2. Fit the reduced model (indicated with the letter R) by excluding the variable in question and all of its associated terms from the model. The error sum of squares for this model is $SS_{\epsilon(R)}$ and the error degrees of freedom are $df_{\epsilon(R)}$.
3. Calculate the test statistic:
$$F = \frac{\left(\frac{SS_{\epsilon(R)} - SS_{\epsilon(F)}}{df_{\epsilon(R)} - df_{\epsilon(F)}}\right)}{\left(\frac{SS_{\epsilon(F)}}{df_{\epsilon(F)}}\right)}$$
The quantity in the numerator is the variance associated with the questionable variable and the quantity in the denominator is the error variance from the full model.
4. If the F statistic exceeds F_α with $df_{\epsilon(R)} - df_{\epsilon(F)}$ numerator and $df_{\epsilon(F)}$ denominator degrees of freedom then reject H_0 and conclude that the variable contributes significantly to the model and should be retained.

Run the *sim5* macro for a 2^5 design with two replicates. Fit the full model including all main effects and two-factor interactions and use this method to determine if x_4 and all its associated terms can be safely dropped from the model.

11. Run *sim5* for a 2^5 full factorial design with a single replicate (Use *2^5f.xls.*). Replace five of the 32 responses with missing values ($*$). Copy the design to another worksheet, delete the rows with the missing values, and construct and interpret the correlation matrix of main effects and two factor interactions. Return to the original worksheet and use the method of Section 9.8 in the textbook to repair the missing values and revise the ANOVA and regression coefficient t and p values.

10

Fractional Factorial Experiments

1. Write out the complete matrix of sixteen runs of the 2^4 experiment design, then indicate which runs correspond to the half fraction with generator $4 = 123$ and which half correspond to the fraction with generator $4 = -123$.

2. The model for a full factorial 2^3 experiment is:

$$y = 660 + 90x_1 - 140x_2 + 55x_3 + 12x_{12} - 6x_{13} + 14x_{23} - 3x_{123}$$

where the coefficients shown are the parameters. If a 2^{3-1}_{III} experiment was built with generator $3 = 12$, then what terms could be modeled and what would the coefficients become?

3. The model for a full factorial 2^4 experiment is:

$$y = 54 - 13x_1 + 14x_2 + 52x_3 - 19x_4 + 2x_{12} - 6x_{13} + 5x_{14} - 1x_{23} + 5x_{24} - 8x_{34}$$

where the coefficients shown are the parameters. If a 2^{4-1}_{IV} experiment was built with generator $4 = 123$, then what terms could be modeled and what would the coefficients become?

4. Select 16 runs for 2^{5-1}_{V} design from the original 32 run 2^5 experiment in Problem 9.3. Analyze the experiment and compare the results to those of the full analysis. How much information is lost and how much is retained?

5. Run the *sim4* macro for two replicates of a 2^4 design and analyze the data. (Start from *2^4f.xls*.) Select one half of the original runs of a 2^{4-1}_{IV} design and analyze this experiment. Refine the two models and compare them.

6. Construct a 2^{5-2}_{III} design and its fold-over design. Use correlation matrices to demonstrate that the two designs individually are resolution III but that their combination is resolution IV.

7. Run the *sim5* macro for one replicate of the 2^{5-1}_{V} design. Starting from the model with all main effects and two factor interactions, refine the model one term at a time by eliminating the weakest term at each step, however, be certain to preserve the hierarchy of the model

terms. At each step, record the model standard error and adjusted coefficient of determination. Plot these two quantities as a function of model degrees of freedom. What happens when you begin dropping significant model terms? Stop when it is clear from the plots which models might be appropriate.

8. Run the *sim5* macro for one and two replicates of the 2_V^{5-1} design. Be sure that the standard deviation is set to $\sigma_\epsilon = 10$. Analyze the data for main effects and two-factor interactions and refine the models to eliminate statistically insignificant terms. How do the steps in the two analyses and the assumptions that you have to make differ between the two experiments? Transcribe the final regression coefficients and their summary statistics into the columns *2^5hr1* and *2^5hr2*, respectively, of the worksheet in *Homework Problems> sim5 Worksheet.xls*. Use the codes suggested on the worksheet to indicate the statistical significance of each regression coefficient.

9. How many replicates of a 2_V^{5-1} design are required to deliver 90% power to detect a different $\delta = 30$ between the two levels of a variable when $\sigma_\epsilon = 40$.?

10. A screening experiment was performed to identify spot welding process variables that affect the diameter of the weld nugget. A 2_{VI}^{6-1} design with one replicate was used. The variables and their levels are shown in the table below and the data are shown in Table 10.1. Analyze the data and make a recommendation on how to improve the spot welding process by increasing the diameter of the weld nugget.

Code	Variable	-1	$+1$
A	Axial Misalignment	no	yes
B	Angular Misalignment	no	yes
C	Edge Weld	no	yes
D	Part Surface Misfit	no	yes
E	Cooling Rate	low	high
F	Electrode Age	New	Ancient

A	B	C	D	E	F	Y
1	1	-1	-1	-1	-1	3.6
-1	-1	-1	-1	-1	-1	6.0
1	1	-1	1	-1	1	2.7
1	1	-1	1	1	-1	2.3
1	-1	-1	-1	1	-1	4.2
-1	-1	-1	1	-1	1	3.8
-1	1	-1	-1	-1	1	5.1
1	1	1	1	1	1	5.2
-1	-1	1	1	1	1	4.7
-1	1	1	-1	1	1	5.5
1	-1	1	-1	-1	-1	7.1
-1	-1	1	-1	-1	1	6.5
-1	1	1	1	-1	1	4.6
-1	1	1	1	1	-1	6.2
1	-1	1	1	-1	1	5.1
1	-1	-1	1	-1	-1	2.5
1	-1	-1	-1	-1	1	3.4
1	1	1	-1	-1	1	5.9
-1	-1	-1	1	1	-1	5.7
-1	-1	1	1	-1	-1	5.0
1	-1	1	-1	1	1	6.0
-1	-1	-1	-1	1	1	5.2
-1	1	-1	-1	1	-1	5.6
-1	1	1	-1	-1	-1	6.6
1	-1	1	1	1	-1	5.7
1	1	1	-1	1	-1	6.6
-1	1	-1	1	-1	-1	5.2
-1	-1	1	-1	1	-1	6.7
1	1	-1	-1	1	1	3.8
1	-1	-1	1	1	1	2.6
-1	1	-1	1	1	1	4.5
1	1	1	1	-1	-1	5.8

TABLE 10.1. Spot Weld Screening Experiment

11

Response Surface Experiments

1. An experiment was performed to study weight gain in pigs using a $CC(2)$ design with three replicates. The pigs were weaned at different body weights (27, 29, 34, 39, and 41 lb) and were put on a feeding schedule that varied relative to the nominal feeding schedule (-14, -10, 0, 10, and 14 %). The experimental responses were the number of days required to reach the 250lb market weight (Day) and the total amount of feed consumed (Feed). The experimental runs and responses are shown in the table below. Analyze the two responses and recommend a strategy to minimize the costs associated with pig production. What factors are not considered in this study that could influence your decision to implement this strategy?

FeedRate	Weight	Day	Feed
-10	29	121	540
-10	29	113	539
-10	29	119	553
10	29	99	532
10	29	98	533
10	29	98	540
-10	39	112	528
-10	39	109	539
-10	39	111	532
10	39	93	530
10	39	94	530
10	39	93	522
-14	34	120	525
-14	34	122	518
-14	34	120	536
14	34	85	534
14	34	89	541
14	34	94	542

FeedRate	Weight	Day	Feed
0	27	110	545
0	27	112	549
0	27	107	570
0	41	98	519
0	41	100	522
0	41	99	533
0	0	106	543
0	0	107	542
0	0	107	515
0	0	102	551
0	0	106	523
0	0	107	518
0	0	105	528
0	0	105	528
0	0	110	542
0	0	109	538
0	0	100	526
0	0	109	540

2. Weight gain in pigs soon to reach their 250 lb market weight was studied as a function of a two component feed composition: corn, which is high in carbohydrates, and soy, which is high in protein. A 3^2 design was used with two replicates. The experimental data are shown in the table below where corn, soy, and 20-day weight gain are all in pounds. Construct a model for weight gain and develop a strategy for managing pig feed. (Warning: Transform the levels of corn and soy into a new pair of variables that are orthogonal before attempting to fit a response surface model.)

Corn	Soy	Weight
84	36	33
98	42	38
112	48	40
96	24	35
112	28	41
128	32	54
108	12	38
126	14	45
144	16	50
84	36	35
98	42	36
112	48	46
96	24	39
112	28	42
128	32	49
108	12	35
126	14	49
144	16	51

3. Run the *sim5* macro for the following experiment designs, analyze the data, refine the models, and transcribe the regression coefficients into the appropriate fields in the worksheet *Homework Problems> sim5 Worksheet.xls*. Be sure that the standard deviation is set to $\sigma_\epsilon = 10$ in each case before you run the macro.

Design	**Start from:**	**Worksheet column:**
one replicate of a 2^5 design with six centers	*2^5f*	*2^5fr1c*
one replicate of a 2_V^{5-1} design with five centers	*2^5h*	*2^5hr1c*
one replicate of a $CC\,(2^5)$ design	*CC5f*	*CC5fr1*
one replicate of a $CC\left(2_V^{5-1}\right)$ design	*CC5h*	*CC5hr1*
one replicate of a $BB\,(5)$ design	*BB5*	*BB5r1*
one replicate of a 3^5 design	*3^5f*	*3^5fr1*

Now that the worksheet is complete:

(a) Do the two level designs capture the essential behavior of the *sim5* process?

(b) Does the addition of center cells to the two level designs help the situation? Is the fix adequate? Can the resulting models be used to make predictions including interpolations within the bounds of the x_i?

(c) Of the true response surface designs, which design gives the best model and why?

(d) Of the true response surface designs, which is the smallest experiment that captures the essential behavior of the *sim5* process?

(e) Describe the advantages and disadvantages of the various families of experiment designs.

4. An experiment must be performed to maximize the strength of spot welds used to join two sheets of 0.021" thick low carbon steel. The primary response, weld shear strength (pounds), is measured by pulling on the welded sheets in opposite directions along a line that passes through the weld in the plane of the sheets until the weld breaks. Since weld strength can be inconvenient to measure, weld nugget diameter (inches) is often used as a surrogate for weld strength. The nugget diameter is measured with electronic calipers after the welded sheets are pulled apart.

   The process engineers responsible for the spot welding process have identified three process factors that they feel substantially determine the spot weld strength: welding time, electrode clamping force, and current. The process engineers feel that these variables have the following safe and potentially useful upper and lower limits:

Variable	**Lower**	**Upper**	**Units**
Time	6	22	*cycles*
Force	100	300	*pounds*
Current	3000	7000	*amps*

   Design an appropriate experiment to study this process and run the experiment from the *weld.xls* simulation in the *Homework Problems* folder. Specify the physical center or $x = 0$ levels of each design variable in the *Center* column and the physical unit step size corresponding to the difference between the $x = 0$ and $x = \pm 1$ levels in the *Step* column. Specify your desired experiment design in columns x_1 through x_3 in the worksheet in coded units and then run the simulation. Analyze the experimental data to determine the operating conditions that maximize the weld strength and that maximize the nugget diameter. Confirm that: 1) the two maximal solutions are obtained under similar process conditions and 2) that nugget diameter is a valid substitute for weld strength.

5. An experiment was performed to study the deflection of simply supported rectangular beams as a function of beam height, beam width, beam span, and load. Rectangular basswood beams were simply supported at their ends and point loaded at the center of the span where beam deflection was measured. A $BB(4)$ design was used with the following variable levels:

Variable	-1	**0**	$+1$	**Units**
A : Height	0.125	0.250	0.375	*inch*
B : Width	0.125	0.250	0.375	*inch*
C : Span	16	19.5	23	*inch*
D : Load	1	2	3	*N*

The data are shown in the following table. The deflections are reported in inches.

A	B	C	D	$Deflection$
1	1	0	0	0.03125
0	0	−1	−1	0.06250
1	−1	0	0	0.15625
0	0	1	−1	0.09375
0	0	−1	1	0.15625
0	0	1	1	0.43750
0	0	0	0	0.15625
−1	−1	0	0	3.00000
−1	1	0	0	0.12500
0	−1	1	0	2.37500
0	−1	−1	0	0.75000
1	0	0	1	0.21875
0	1	1	0	0.09375
0	0	0	0	0.21875
−1	0	0	1	0.50000
0	1	−1	0	0.06250
−1	0	0	−1	0.18750
1	0	0	−1	0.06250
0	1	0	−1	0.03125
−1	0	−1	0	0.21875
0	1	0	1	0.09375
−1	0	1	0	0.62500
0	0	0	0	0.25000
0	−1	0	1	2.25000
1	0	1	0	0.21875
0	−1	0	−1	0.75000
1	0	−1	0	0.06250

Analyze this experiment. How well does the model fit the data? What should the deflection be when the load or span are zero and what does your model predict? What could be wrong?

6. The theoretical equation for the maximum deflection Δ of a centrally-loaded simply-supported rectangular beam is given by:

$$\Delta = \frac{1}{4E}\frac{FL^3}{wh^3} \tag{11.1}$$

where E is the elastic modulus of the beam material, F is the load, L is the span, w is the beam width, and h is the beam height. Fit a model of this form to the data from Problem 11.5 and compare the performance of the two models. Why do some of the exponents of variables deviate from those indicated in the theoretical equation? Given the implications of the theoretical formula for deflection, how would you design a new experiment to study this process?

7. Run the *sim5* macro (*CC5f.xls*) for a $CC\left(2^5\right)$ design with $\sigma_\epsilon = 10$ and run the specified analyses on the specified subsets of the complete experiment. Refine each model and make a table of the regression coefficients. Use your table to compare the performance of the different designs and analyses.

   (a) The 32 runs of the 2^5 design with main effects and two-factor interactions.

   (b) The 32 runs of the 2^5 design plus the center points with main effects and two-factor interactions.

   (c) The 32 runs of the 2^5 design plus the center points with main effects, two-factor interactions, and the generic curvature term.

   (d) The complete $CC\left(2^5\right)$ design with main effects and two-factor interactions.

   (e) The complete $CC\left(2^5\right)$ design with main effects, two-factor interactions, and the generic curvature term.

   (f) The complete $CC\left(2^5\right)$ design with main effects, two-factor interactions, and quadratic terms.

   (g) The center points and star points with main effects and quadratic terms.

8. The five level central composite designs are sometimes referred to as *circumscribed* central composite designs. One of the complications of these designs is that it is frequently difficult to obtain the required five levels of all of the design variables. A variation on central composite designs, called central composite *inscribed* or *face-centered* designs, places the star points at the same ± 1 levels as the 2^k part of the design. This reduces the number of levels required for each design variable from five to three.

   Construct the five variable inscribed central composite design from the circumscribed $CC\left(2_V^{5-1}\right)$ design by changing the $\pm\eta$ star point levels to ± 1, respectively. Calculate the correlation matrices for both designs including main effects, two-factor interactions, and quadratic terms. How do the two designs compare with respect to their ability to resolve effects? What do these observations indicate about the inscribed designs if their primary advantage over simpler designs, like the 2^k plus centers designs, is their ability to resolve curvature in the response?

9. Determine the number of replicates and total number of runs required for 3^5, $BB\left(5\right)$, and $CC\left(2_V^{5-1}\right)$ designs if the regression coefficients of main effects (in ± 1 coded units) must be resolved to within $\delta = \pm 5$ with 95% confidence. The standard error of the model is expected to be $\sigma_\epsilon = 50$. Use $\alpha = 0.05$ and assume that the same physical levels are used for the ± 1 levels in all three designs.

10. Fit a response surface model to the battery life data from Problem 1.7.

11. Match each experiment design to its description.

Answer	Description
	Detects but can't resolve curvature
	1/8th fraction resolution IV design
	Subset of a 3^k design with extra center cells
	Can include up to 11 two-level study variables
	Response surface design, almost never used.
	Resolves main effects and interactions up to the k-factor interaction
	Five-level three-variable experiment
	Saturated model with main effects and all two-factor interactions
	Two levels of each of k variables

(a) 3^k

(b) 2^k

(c) $CC\,(2^3)$

(d) 2^{7-3}

(e) $PB\,(12)$

(f) 2^k plus centers

(g) $BB\,(k)$

(h) 2^{5-1}