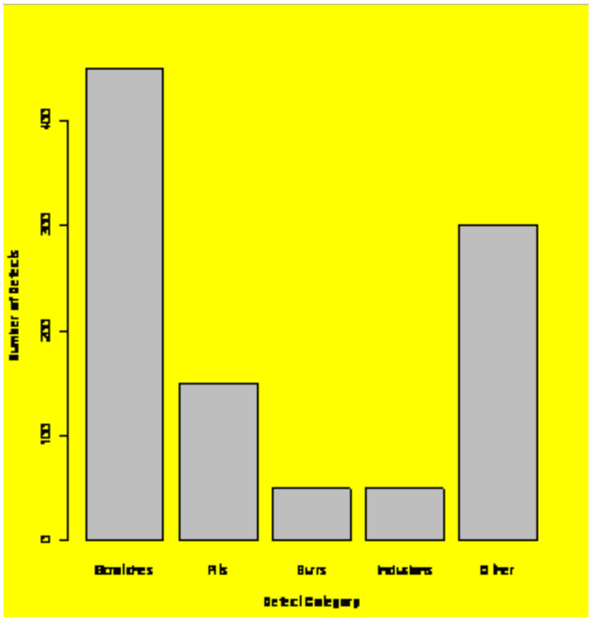
```
#################################################################################

#                          R Code to Supplement
#                   Design of Experiments with MINITAB
#                           by Paul Mathews
#                   published by ASQ Quality Press (2004)

#################################################################################

#This file contains R code that reproduces the examples from Design of Experiments
#with MINITAB by Mathews. In some cases, alternative and extra methods are given where
#R has special capabilities.

#The code contained here was run on R Version 2.0.1 and produces output comparable to
#that from MINITAB V14. There are probably more accurate, efficient, or better ways of
#producing the same or similar output in R than the methods shown here, but the methods
#shown work.

#If you find any mistakes or have recommendations on how to improve this document,
#please communicate them to me by e-mail at the address below.

#Paul Mathews and MM&B Inc. take no responsibility for the accuracy, stability, use,
#or misuse of any of the R code contained here.

#Copyright © 2005 Paul Mathews

#Paul Mathews, President
#Mathews Malnar and Bailey, Inc.
#E-mail: paul@mmbstatistical.com
#Web: www.mmbstatistical.com

#Rev. 4/20/05 (PGM)
#Rev. 5/18/05 (PGM) Added TukeyHSD p value calculations from library(multcomp).
#Rev. 7/19/05 (PGM) Added command to change lattice/trellis background from gray to white
 in Example 1.3.
```

```
###################################################################################

#              R Conventions

###################################################################################

#Command prompt: R commands are submitted by typing them at the command prompt ">".
#Two or more commands separated by semicolons may appear on the same line. If an R
#command is too long for one line, hit <Enter> and R will allow you to continue the
#command on the next line at the "+" prompt. For example:

#       > help(
#       + lm)                 #Find help for the lm function.

#Several commands that need to be considered together may be collected within braces
#{} which may be separated over several lines.

#Objects: Data in R and the output of R analyses are all "objects" that have properties
#defined within R. For example, the R command:

#       > Y.lm = lm(Y~X)

#fits a linear model for Y as a function of X. The output of the lm function is assigned
#to the new object Y.lm, but the lm function by itself doesn't create any visible output.
#There are other functions which extract information/output from Y.lm: summary(), anova(),
#coefficients(), residuals(), plot(), etc. The results from each of these functions are
#themselves objects.

#Object names: R commands and object names are case sensitive. For example, the command
#"anova" is different from "Anova". Object names must start with a letter and can contain
#numbers and limited special characters. Periods are commonly used as delimitors in object
#names, e.g. y.lm.residuals.

#Object class: Objects have different classes that have different properties. When objects
#are passed to a function, they must be compatible with the expected class required by the
#function. Many errors in R commands are caused by mismatched object classes.

#Assignment operations: Objects are assigned values using the R assignment operators "="
#or "<-". For example, y.lm = lm(y~x) assigns the output from the lm function to the object
#y.lm. The class of y.lm will be determined by the class of lm's output.

#Boolean operations: Boolean operations are used to control branching in if statements.
#They must appear inside of parentheses. The results from Boolean operations are either
#TRUE or FALSE. The Boolean "equals" operator is two equals signs: "==". For example, the
#Boolean operation (x1==5) is TRUE if x1 equals 5 and FALSE if it's not. The Boolean "not
#equals" operator is "!=", as in (x1!=5). The Boolean "or" operator is "||", as in (A || B).

#Missing values: Missing values in data sets are indicated with "NA". For example:

#       > x1 = c(1,3,2,3,1,1,NA,2,2)

#Comments: Anything typed after a pound (#) symbol on the command line is interpreted as
#a comment by R.
```

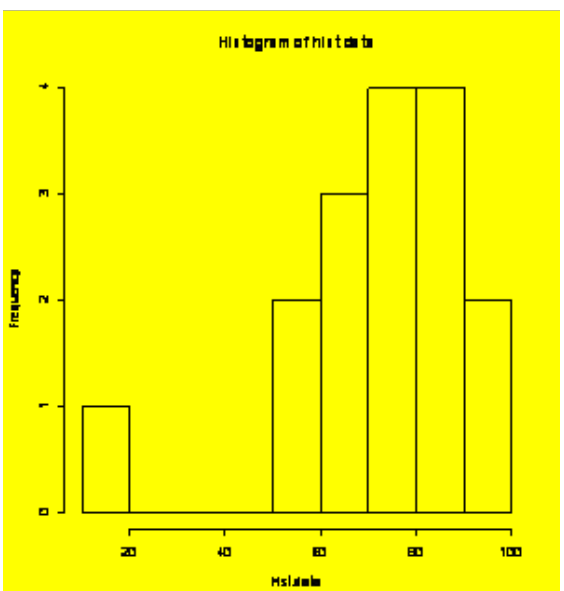
```
##################################################################################

#              R Utility Functions

##################################################################################

#The following utility functions are basic to operations in R. Knowledge of their usage
#is assumed throughout the material that follows. This list does not include any
#analysis functions.

#The functions are given in alphabetical order. Use help() and help.search() to find
#more details about a function or topic.

#Function                              #Description
-----------------------------------------------------------------------------------------
#apropos("for")                        #Returns all objects that contain the indicated string.

#attach(Y.data)                        #Puts the data in object Y.data on R's search list.

#x1=c(1,1,1,2,2,2,3,NA,3)              #c() is the concatenate function - assigns x1 the
                                       #values 1,...,3. NA is a missing value.

#Y.des.mat=cbind(des.mat,Y)            #Appends columns of two objects into a new object

#class(y.lm)                           #Returns the class of the object y.lm.

#data.entry(x=c(NA))                   #Opens a spreadsheet environment for data entry for
                                       #object x.

#y.data.frame=data.frame(y,x1,x2,x3)  #Creates a data frame y.data.frame of y, ...

#detach(y.data.frame)                  #Remove y.data.frame from the search path.

#ID=factor(ID)                         #Changes ID into a factor, e.g. for a predictor in
                                       #ANOVA.

#for (i in 1:5) {}                     #Loop from i=1 to 5, do the operation(s) in {} each
                                       #time through the loop.

#x1=gl(3,2,24)                         #Generate a list of integers 1 to 3, repeated twice
                                       #each, for a total of 24 values.

#help(lm)                              #Find help files for the lm function. In some cases,
                                       #the argument must appear in quotes, e.g. help("if").

#help.search("residuals")              #Search the help files for the word "residuals".

#if (x==1) {y=5} else {y=0}            #An if/else statement. This statement can be split on
                                       #several lines, but "} else {" must appear in the same
                                       #line.

#length(x)                             #Returns the length of the vector x, i.e. its number
                                       #of observations.

#letters[1:5]                          #Create a list of lower case letters "a", ..., "e".
#LETTERS[1:5]                          #Upper case letters "A", ..., "E".

#library(multcomp)                     #Load the package multcomp.

#ls()                                  #List the current objects.

#load("c:/R/my.Rdata")                 #Loads the data in the indicated file. See save().

#names(Y.data.frame)                   #Prints the names of the variables in the data frame.

#Yby3 = Y[order(Y[,3]),]               #Orders the data frame Y by its third column.

#par(mfrow=c(2,2))                     #par sets graphics parameters, mfrow makes figures
                                       #arranged in rows (2) and columns (2).

#print(y)                              #Print the object y, equivalent to just typing "y" at
                                       #the command prompt.

#quit()                                #Quit the R program, equivalent to File> Exit.

#Y.DOE=rbind(replicate1,replicate2)    #Appends rows from one object onto another.

#read.table("c:/R/junk.dat",header=TRUE,sep="")       #Reads white-space-separated data from
                                                      #file junk.dat using a one-line header.

#x.rep=rep(x,4)                        #Repeat the contents of x four times, store result in x.rep.

#rm(x,y)                               #Remove (delete) objects x and y from the R session.
```

```
#save(x, y, file = "my.Rdata")        #Save objects x and y in file my.Rdata. See load().

#save.image()                         #Save objects, packages, etc., i.e. the whole R environment.

#x=seq(10,30,2)                       #Generate a sequence of numbers from 10 to 30 in steps of size 2

#sink("output.txt")                   #Copies the text output from R to the indicated file.

#source("c:/R/mycode.R")              #Runs the R code in the indicated file.
```

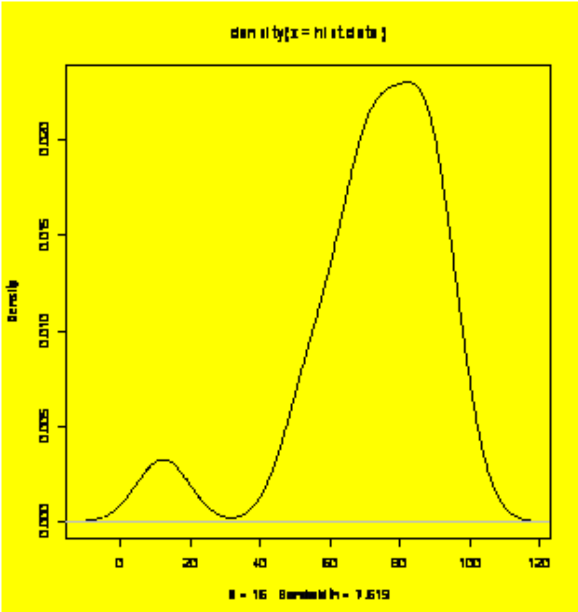
```
################################################################################

#              CHAPTER 1: Graphical Presentation of Data

################################################################################

### Example 1.1 (p. 2) Bargraph of defects data.
defect.freq=c(450,150,50,50,300)
defect.names=c("Scratches","Pits","Burrs","Inclusions","Other")
barplot(defect.freq,names.arg=defect.names,xlab="Defect Category",ylab="Number of Defects")
```




```
### Example 1.2 (p. 3) Histogram of example data with forced categories.
hist.data=c(52,88,56,79,72,91,85,88,68,63,76,73,86,95,12,69)
hist(hist.data,breaks=c(10,20,30,40,50,60,70,80,90,100))
```

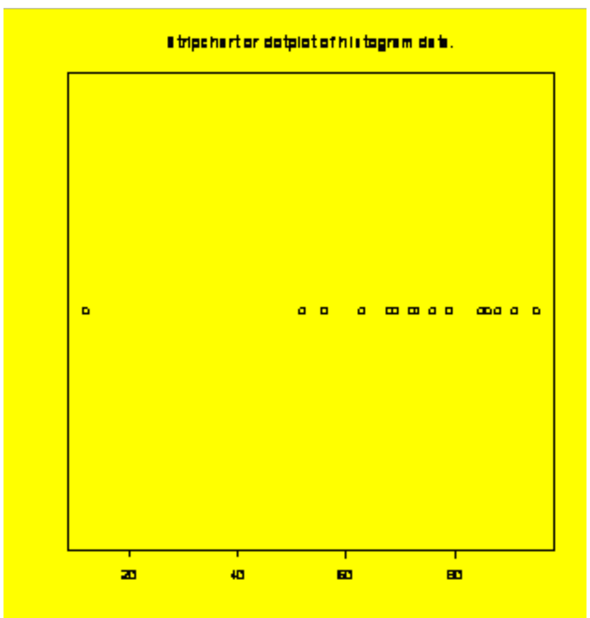



```
### Extra: Density plot.
plot(density(hist.data))
```



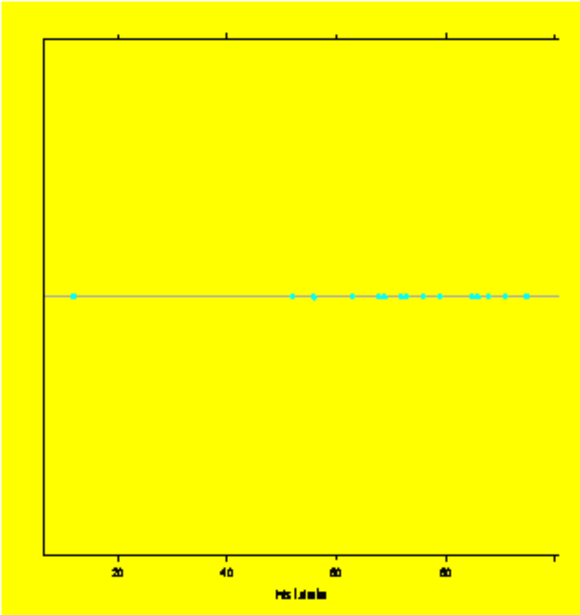
Example 1.3 (p. 4) Dotplot (or stripchart in R).

```
stripchart(hist.data);title("Stripchart or dotplot of histogram data.")#Add the title to the dotplot
```



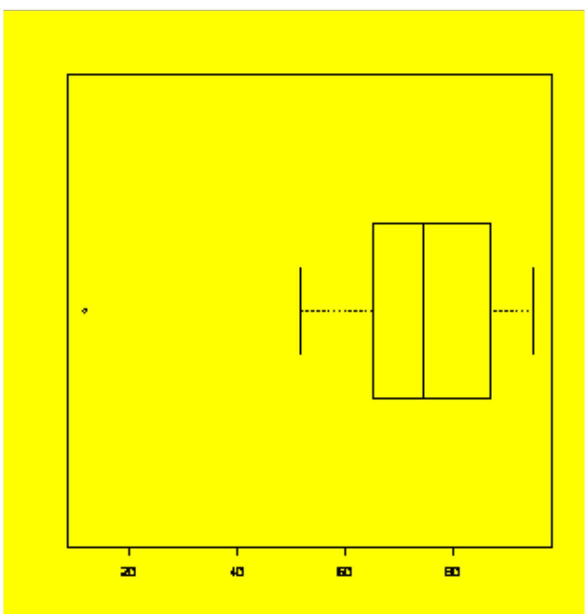
Alternative dotplot using lattice package:

```
library(lattice)
trellis.par.set(background=0)                          #Change background from default gray (1) to white (0)
dotplot(hist.data)
```

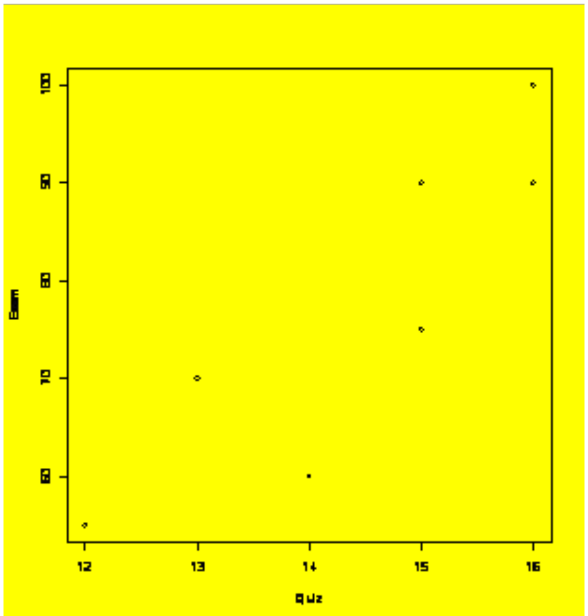



```
### Example 1.4 (p. 4) Stem and leaf plot.
stem(hist.data)

   The decimal point is 1 digit(s) to the right of the |

  0 | 2
  2 |
  4 | 26
  6 | 3892369
  8 | 568815
```

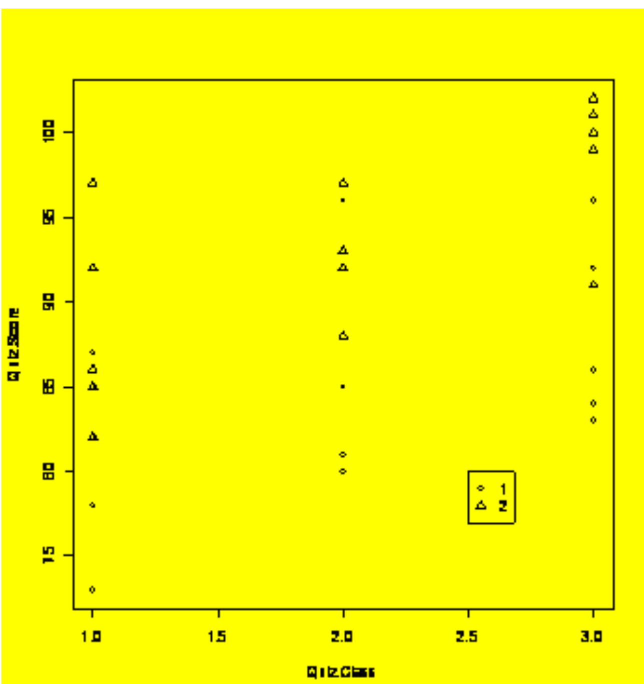
```
### Example 1.5 (p. 6) Boxplot.
boxplot(hist.data,horizontal=TRUE)
```



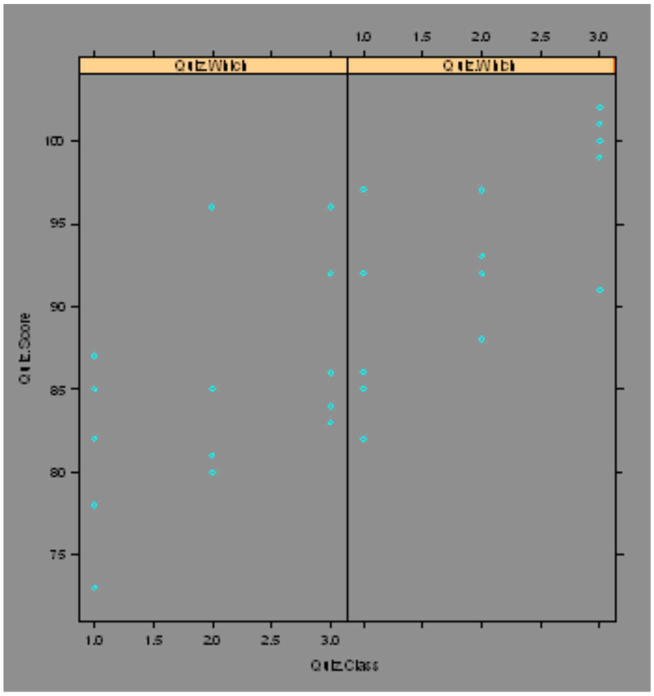

```
### Example 1.6 (p. 7) Scatter plot.
Quiz=c(12,14,13,15,15,16,16)
Exam=c(55,60,70,75,90,90,100)
plot(Quiz,Exam)
```



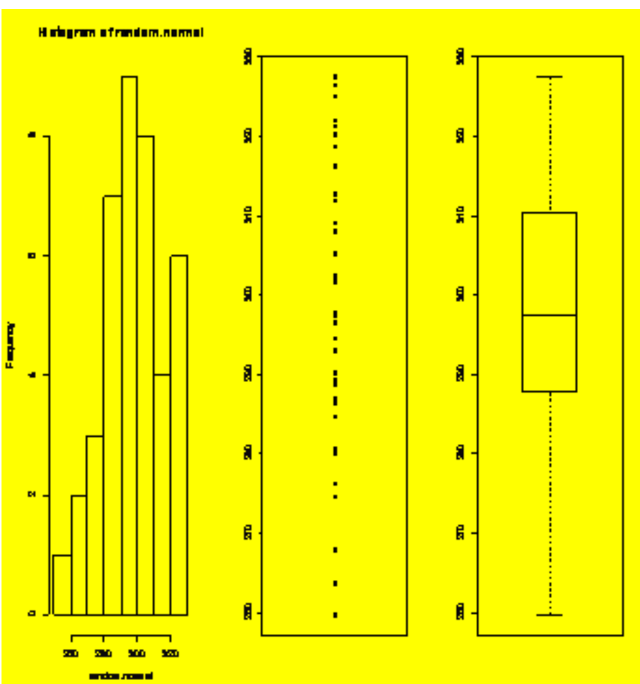

```
### Example 1.7 (p. 8) Multi-vari chart.
Quiz.Student=c(1,2,3,4,5,1,2,3,4,5,1,2,3,4,5,1,2,3,4,5,1,2,3,4,5,1,2,3,4,5)        #Alternatively:
Quiz.Student=gl(5,1,30)
Quiz.Class=c(1,1,1,1,1,1,1,1,1,1,2,2,2,2,2,2,2,2,2,2,3,3,3,3,3,3,3,3,3,3)          #Quiz.Class=gl(3,10,30)
Quiz.Which=c(1,1,1,1,1,2,2,2,2,2,1,1,1,1,1,2,2,2,2,2,1,1,1,1,1,2,2,2,2,2)          #Quiz.Which=gl(2,5,30)
Quiz.Score=c(87,82,78,85,73,86,92,82,85,97,81,85,85,80,96,97,93,92,88,88,84,
96,86,92,83,100,91,102,99,101)
Quiz=data.frame(Quiz.Student,Quiz.Class,Quiz.Which,Quiz.Score)                     #Makes the data frame
plot(Quiz.Score~Quiz.Class,pch=Quiz.Which)
### After the plot is created, add the legend to it with the following command:
legend(2.5,80,legend=c(1,2),pch=c(1,2))                                            #Adds legend at position(2.5,80)
```




```
### Example 1.7 (p. 8) Alternative multi-vari chart using lattice package:
library(lattice)
xyplot(Quiz.Score~Quiz.Class|Quiz.Which)                              #Plots Score vs. Class in two panels defined by
Which
```

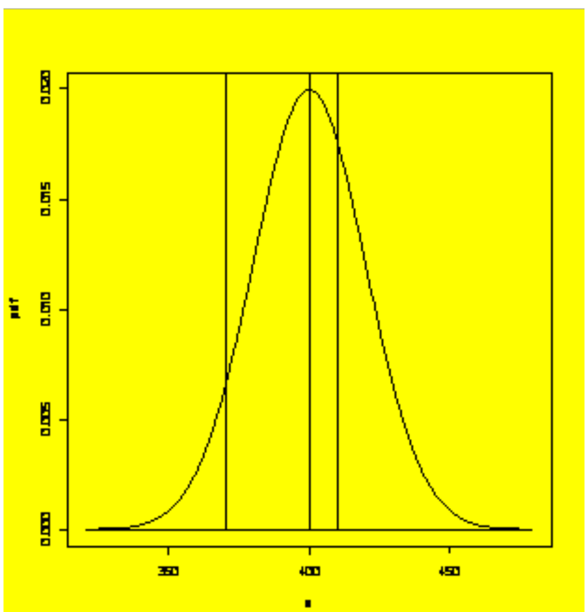



```
### Example 1.9 (p. 16) Script that creates and plots random normal data.
random.normal=rnorm(40,300,20)                              #(sample size, mean, standard deviation)
par(mfrow=c(1,3))                                           #Three graphs on one page, 1 row by 3
columns
hist(random.normal)
stripchart(random.normal,vertical=TRUE)
boxplot(random.normal)
```



```
par(mfrow=c(1,1)                                            #Reset the graphics display to 1 row and 1
column


########################################################################

#             CHAPTER 2: Descriptive Statistics

########################################################################

### Example 2.15 (p. 34) Calculating statistics from sample data.
```

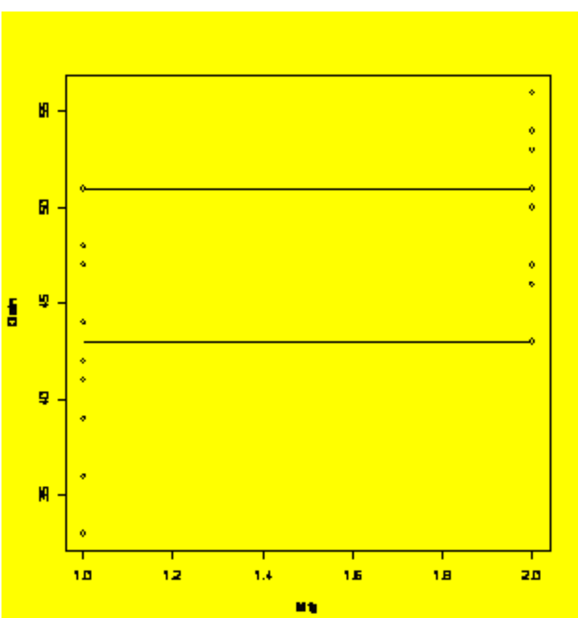
```
Example.Data=c(16,14,12,18,9,15)
length(Example.Data)                          #Sample size
[1] 6

mean(Example.Data)
[1] 14

sd(Example.Data)
[1] 3.162278

range(Example.Data)#Reports min and max values
[1]  9 18

diff(range(Example.Data))#Sample range
[1] 9

summary(Example.Data)#Common summary statistics
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
   9.00   12.50   14.50   14.00   15.75   18.00


### Example 2.16 (p. 35) Calculating and plotting the normal probability density function.
x=seq(320,480,1)                              #The array of x values
pdf=dnorm(x,400,20)                           #The corresponding pdf
y.max=1.1*max(pdf)                            #Upper limit for y axis
plot(x,pdf,type="l")                          #Create the plot, type is "l"ine
### Now add the requested reference lines
xref=c(370,370)
yref=c(0,y.max)
lines(xref,yref)                              #Reference line at x=370
xref=c(400,400)
lines(xref,yref)                              #Reference line at x=400
xref=c(410,410)
lines(xref,yref)                              #Reference line at x=410
xref=range(x)
yref=c(0,0)                                   #Reference line at y=0
lines(xref,yref)
```



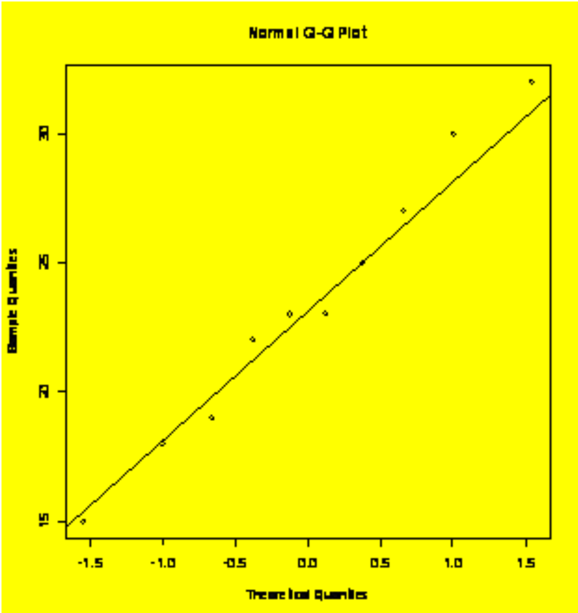

```
##################################################################################

#              CHAPTER 3: Inferential Statistics

##################################################################################

# Many of the problems in this chapter make use of summarized data. For example, the
# sample mean, standard deviation, and sample size are given instead of the raw data
# for problems in calculating confidence intervals and performing hypothesis tests.
# Normally one would use R to perform all of these operations. To fill these data gaps
# and demonstrate the use of R, the affected examples shown here use data sets from
# other examples in the book.

### Example 3.2 (p. 42) Confidence interval for the population mean (sigma known).
### Example: For the data from Example 3.24, find the 95% confidence interval for the population
### mean assuming that the population standard deviation is known to be sigma = 5.
one.sample.z.ci=function(x,sigma,conf=0.95)          #Function to find the one-sample two-sided z CI
```

```
{
zhalfalpha=-qnorm((1-conf)/2);SE=sigma/sqrt(length(x))
mean(x)+c(-zhalfalpha*SE,zhalfalpha*SE)              #Here's the CI calculation
}
Y=c(22,25,32,18,23,15,30,27,19,23)                   #Data from Example 3.24
one.sample.z.ci(Y,5)                                 #Call the function with sigma=5, 95% default confidence

[1] 20.30102 26.49898


### Example 3.6 (p. 47) Hypothesis test for one sample location (sigma known).
### Example: Test the data from Example 3.24 to see if the population mean is different from
### mu = 20 assuming that the population standard deviation is known to be sigma = 5.
one.sample.z.test=function(x,sigma,mu0)              #Function for the one-sample two-sided z test
{
z=(mean(x)-mu0)/(sigma/sqrt(length(x)))
2*pnorm(-abs(z))
}
Y=c(22,25,32,18,23,15,30,27,19,23)                   #Data from Example 3.24
one.sample.z.test(Y,5,20)                            #Function reports the two-sided p value

[1] 0.03152763


### Example 3.10 (p. 54) Hypothesis test for one sample location (sigma unknown).
### Example: Test the data from Example 3.24 to see if the population mean is different from mu = 20.
Y=c(22,25,32,18,23,15,30,27,19,23)                   #Data from Example 3.24
t.test(Y,mu=20)                                      #Reports the p value and CI

        One Sample t-test

data:  Y
t = 2.0223, df = 9, p-value = 0.07385
alternative hypothesis: true mean is not equal to 20
95 percent confidence interval:
 19.59670 27.20330
sample estimates:
mean of x
     23.4


### Example 3.11 (p. 55) Confidence interval for the population mean (sigma unknown).
### Example: For the data from Example 3.24, determine the 95% confidence interval for the population mean.
Y=c(22,25,32,18,23,15,30,27,19,23)                   #Data from Example 3.24
t.test(Y)                                            #Reports the p value for mu0=0 and the CI

        One Sample t-test

data:  Y
t = 13.9181, df = 9, p-value = 2.158e-07
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 19.59670 27.20330
sample estimates:
mean of x
     23.4


### Example 3.12 (p. 57) Hypothesis test for two samples location (sigmas unknown but equal).
### Example: Test the data from Example 3.20 for a difference between the population means
### assuming that the population variances are equal.
Mfg=gl(2,10,20)
Gain=c(44,41,48,33,39,51,42,36,48,47,51,54,46,53,56,43,47,50,56,53)
t.test(Gain~Mfg,var.equal=TRUE)                      #Equal variance assumption should be tested

        Two Sample t-test

data:  Gain by Mfg
t = -3.4867, df = 18, p-value = 0.002633
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -12.820435  -3.179565
sample estimates:
mean in group 1 mean in group 2
           42.9            50.9

t.test(Gain~Mfg)                                     #Welch's method is preferred

        Welch Two Sample t-test

data:  Gain by Mfg
t = -3.4867, df = 16.776, p-value = 0.002871
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -12.845777  -3.154223
```

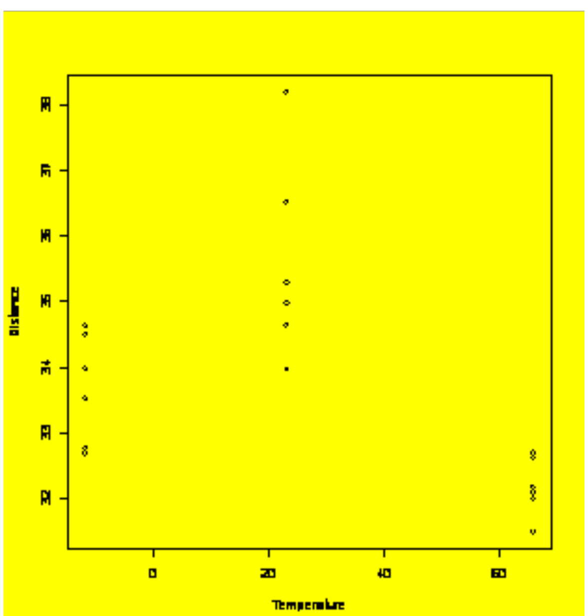
```
sample estimates:
mean in group 1 mean in group 2
           50.9


### Example 3.13 (p. 60) Paired sample t test.
x1=c(44,62,59,29,78,79,92,38)
x2=c(46,58,56,26,72,80,90,35)
x=c(x1,x2)
ID=gl(2,8,16)
t.test(x~ID,paired=TRUE)

        Paired t-test

data:  x by ID
t = 2.443, df = 7, p-value = 0.04456
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 0.07221536 4.42778464
sample estimates:
mean of the differences
                   2.25

### Note: the p-value reported in the book is incorrect. The correct p-value is
### given by: P(-2.443 < t < 2.443; df=7) = 0.04456


### Example 3.14 (p. 64) Chi-square test for one population variance.
### Example: Test the data from Example 3.24 to determine if the population variance
### is larger than sigma = 3.
chisq.p=function(x,sigma0,alt)
{
df=length(x)-1
chisq.statistic=df*var(x)/sigma0^2
if (alt==1) {                              #Right tail test for Ho: sigma > sigma0
pchisq(chisq.statistic,df,lower.tail=FALSE)
} else {                                   #Left tail test for Ha: sigma < sigma0
pchisq(chisq.statistic,df,lower.tail=TRUE)}
}
Y=c(22,25,32,18,23,15,30,27,19,23)
chisq.p(Y,3,1)

[1] 0.0008607428


### Example 3.20 (p. 70) Tukey's quick test.
Mfg=c("A","A","A","A","A","A","A","A","A","A","B","B","B","B","B","B","B","B","B","B")
Gain=c(44,41,48,33,39,51,42,36,48,47,51,54,46,53,56,43,47,50,56,53)
Mfg.Gain=data.frame(Mfg,Gain)
plot(Mfg.Gain)
lines(c(1,2),rep(max(subset(Mfg.Gain$Gain,Mfg=="A")),2))
lines(c(1,2),rep(min(subset(Mfg.Gain$Gain,Mfg=="B")),2))
```



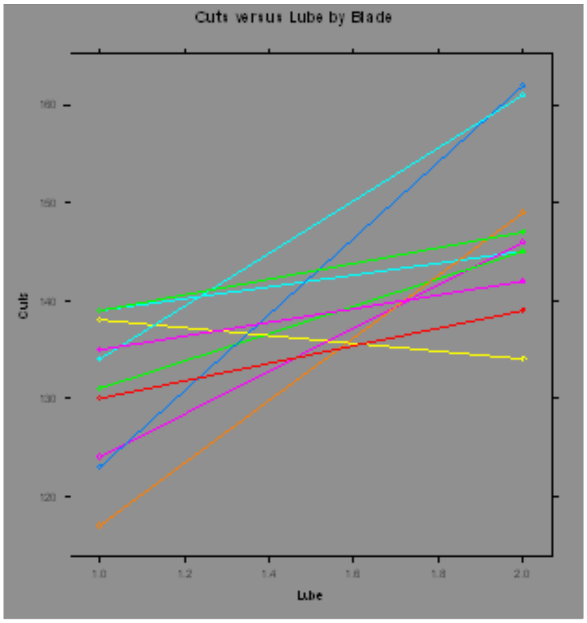

```
### Example 3.24 (p. 77) Normal probability plot.
Y=c(22,25,32,18,23,15,30,27,19,23)
qqnorm(Y);qqline(Y)                  #Creates the normal plot and a line through Q1 and Q3
```



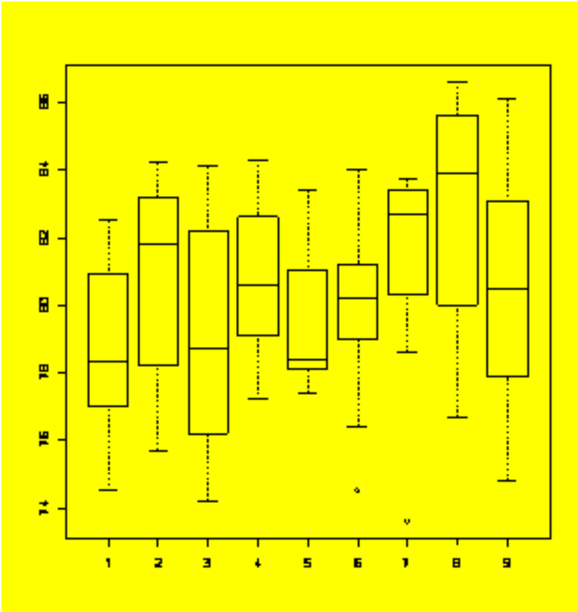

```
shapiro.test(Y)                       #Shapiro-Wilk quantitative test for normality

  Shapiro-Wilk normality test

data:  Y
W = 0.9793, p-value = 0.9613


##################################################################################

#              CHAPTER 4: DOE Language and Concepts

##################################################################################

### Example 4.9 (p. 111) Golf ball flight distance as a function of temperature.
Distance=c(31.5,32.7,33.98,32.1,32.78,34.65,32.18,33.53,34.98,32.63,33.98,35.3,32.7,34.64,36.53,32.0,34.5,38.2)
Temperature=rep(c(66,-12,23),6)
plot(Distance~Temperature)
```




```
### Example 4.12 (p. 4.12) Analysis of saw blade cuts versus lubricant.
Blade=c(5,1,4,9,3,2,3,10,2,6,7,9,8,6,5,8,1,7,4,10)
Lube=c(2,2,1,1,2,1,1,2,2,2,1,2,1,1,1,2,1,2,2,1)
Cuts=c(162,145,117,135,145,124,131,147,146,134,130,142,134,138,123,161,139,139,149,139)
Cuts.data = data.frame(Blade,Lube,Cuts)
library(lattice)
xyplot(Cuts~Lube,groups=Blade,type = "b",main="Cuts versus Lube by Blade",data=Cuts.data)
```



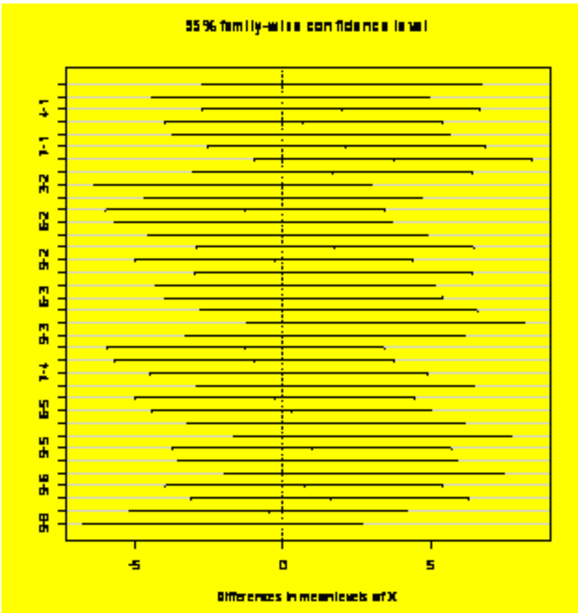

```
Cuts.data.1 = subset(Cuts.data,Lube==1)                    #Subset of the first lube (LAU-003)
Cuts.data.2 = subset(Cuts.data,Lube==2)                    #Subset of the second lube (LAU-016)
Cuts.data.1=Cuts.data.1[order(Cuts.data.1[,1]),]           #Ordered by blade
Cuts.data.2=Cuts.data.2[order(Cuts.data.2[,1]),]
t.test(Cuts.data.1$Cuts,Cuts.data.2$Cuts,paired=TRUE)      #Paired sample t test

        Paired t-test

data:  Cuts.data.1$Cuts and Cuts.data.2$Cuts
t = -3.7482, df = 9, p-value = 0.004568
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -25.656582  -6.343418
sample estimates:
mean of the differences
                    -16


##################################################################################

#              CHAPTER 5: Experiments for One-Way Classifications

##################################################################################

### Example 5.9 (p. 169) ANOVA for a one-way classification with nine treatments.

Y=c(80.9,78.3,77.8,76.6,82.2,74.5,80.5,77,82.5,78.2,81.8,83.5,84.2,75.7,81.4,
78,81.9,83.2,76.2,78.7,79.5,75.3,82.2,78.7,74.2,84.1,83.7,80.6,84.3,80.5,
77.2,82.6,79.1,83.7,81.9,77.9,78.3,83.1,78.9,83.4,81,77.8,77.4,78.4,78.1,
74.5,79,79.7,83.1,76.4,80.2,80.9,81.2,84,83.7,80.3,80.8,83.6,83.4,78.6,
82.8,73.6,82.7,86.6,83.6,85.6,83.9,86,77,80,84.2,76.7,83.1,77.9,77.9,79.9,
83.7,80.5,81.4,74.8,86.1)
X=gl(9,9,81)
boxplot(Y~X)                                 #Boxplots
```



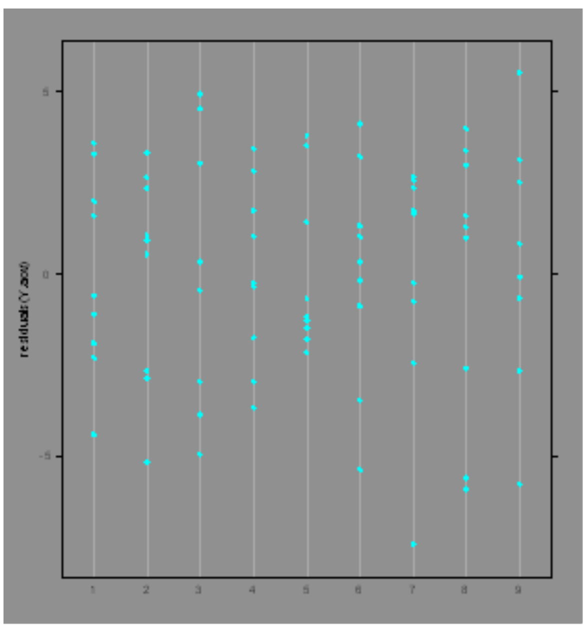

```
Y.aov=aov(Y~X)                                #Perform the ANOVA
summary(Y.aov)                                #Report the ANOVA table

            Df Sum Sq Mean Sq F value Pr(>F)
X            8  93.86   11.73  1.2201 0.2998
Residuals   72 692.34    9.62

aggregate(Y,list(X),FUN=mean)                 #Report the Y means by X

  Group.1        x
1       1 78.92222
2       2 80.87778
3       3 79.17778
4       4 80.86667
5       5 79.60000
6       6 79.88889
7       7 81.05556
8       8 82.62222
9       9 80.58889

TukeyHSD(Y.aov)                               #Report the Tukey HSD CIs

  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = Y ~ X)

$X
           diff        lwr      upr
2-1  1.95555556 -2.7193408 6.630452
3-1  0.25555556 -4.4193408 4.930452
4-1  1.94444444 -2.7304520 6.619341
5-1  0.67777778 -3.9971186 5.352674
6-1  0.96666667 -3.7082297 5.641563
7-1  2.13333333 -2.5415631 6.808230
8-1  3.70000000 -0.9748964 8.374896
9-1  1.66666667 -3.0082297 6.341563
3-2 -1.70000000 -6.3748964 2.974896
4-2 -0.01111111 -4.6860075 4.663785
5-2 -1.27777778 -5.9526742 3.397119
6-2 -0.98888889 -5.6637853 3.686008
7-2  0.17777778 -4.4971186 4.852674
8-2  1.74444444 -2.9304520 6.419341
9-2 -0.28888889 -4.9637853 4.386008
4-3  1.68888889 -2.9860075 6.363785
5-3  0.42222222 -4.2526742 5.097119
6-3  0.71111111 -3.9637853 5.386008
7-3  1.87777778 -2.7971186 6.552674
8-3  3.44444444 -1.2304520 8.119341
9-3  1.41111111 -3.2637853 6.086008
5-4 -1.26666667 -5.9415631 3.408230
6-4 -0.97777778 -5.6526742 3.697119
7-4  0.18888889 -4.4860075 4.863785
8-4  1.75555556 -2.9193408 6.430452
9-4 -0.27777778 -4.9526742 4.397119
6-5  0.28888889 -4.3860075 4.963785
7-5  1.45555556 -3.2193408 6.130452
```

```
8-5  3.02222222 -1.6526742 7.697119
9-5  0.98888889 -3.6860075 5.663785
7-6  1.16666667 -3.5082297 5.841563
8-6  2.73333333 -1.9415631 7.408230
9-6  0.70000000 -3.9748964 5.374896
8-7  1.56666667 -3.1082297 6.241563
9-7 -0.46666667 -5.1415631 4.208230
9-8 -2.03333333 -6.7082297 2.641563
```

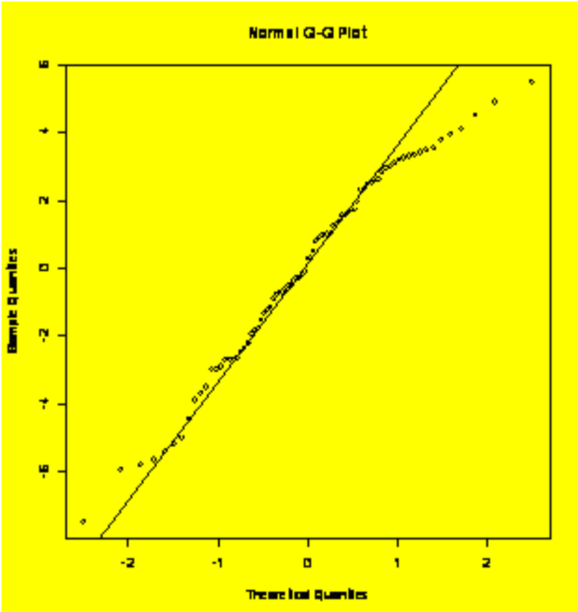
```
plot(TukeyHSD(Y.aov))                         #Plot the CIs
```



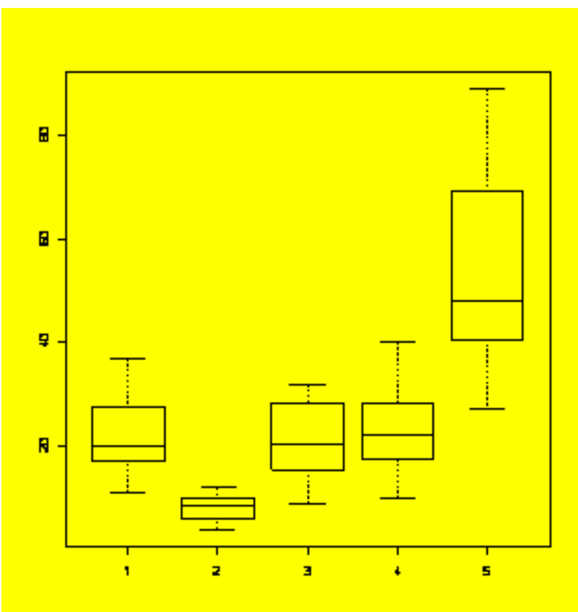

```
dotplot(residuals(Y.aov)~X)                   #Dotplot of residuals
```



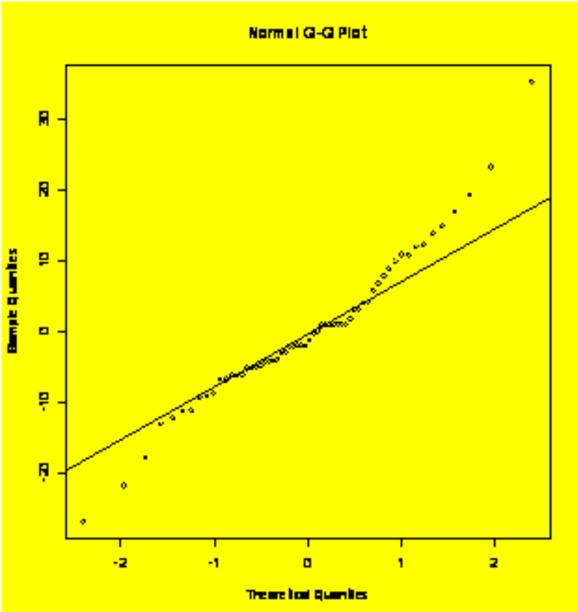
```
qqnorm(residuals(Y.aov)); qqline(residuals(Y.aov))   #Normal plot of residuals
```



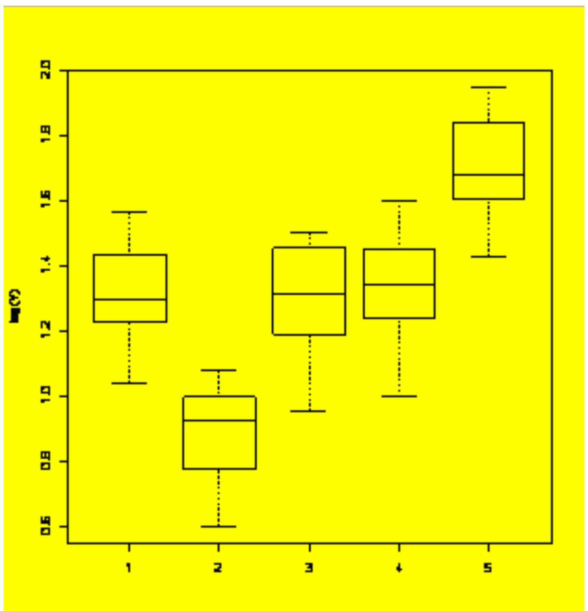

```
### Example 5.11 (p. 180) ANOVA of log-transformed data.
Y=c(31,36,11,24,37,16,18,20,18,20,13,23,6,9,11,9,6,8,11,5,12,4,9,6,10,
15,21,9,29,32,28,27,16,16,20,32,35,19,17,24,18,20,33,24,40,24,10,14,45,36,
49,32,47,89,47,27,58,73,66,77)
X=gl(5,12,60)
boxplot(Y~X)                                 #Check the boxplots - trouble!
```



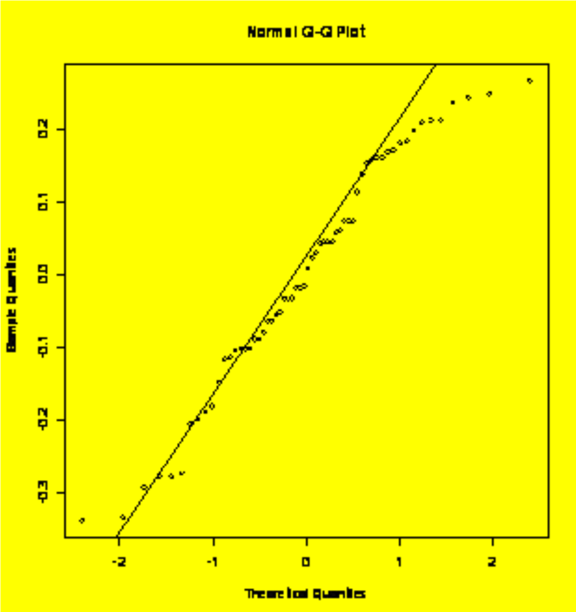
```
Y.aov=aov(Y~X)                                       #Do the ANOVA anyway
qqnorm(residuals(Y.aov)); qqline(residuals(Y.aov))   #Check the ANOVA residuals - trouble!
```



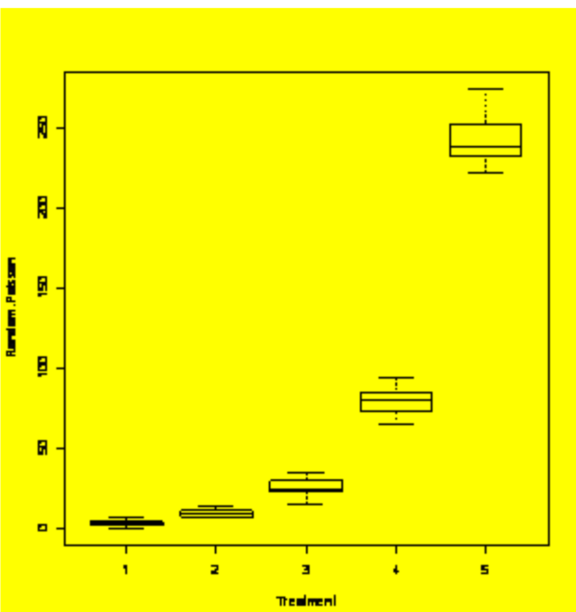

```
boxplot(log10(Y)~X,ylab="log(Y)")          #Boxplot of log transformed Y values - looks better!
```



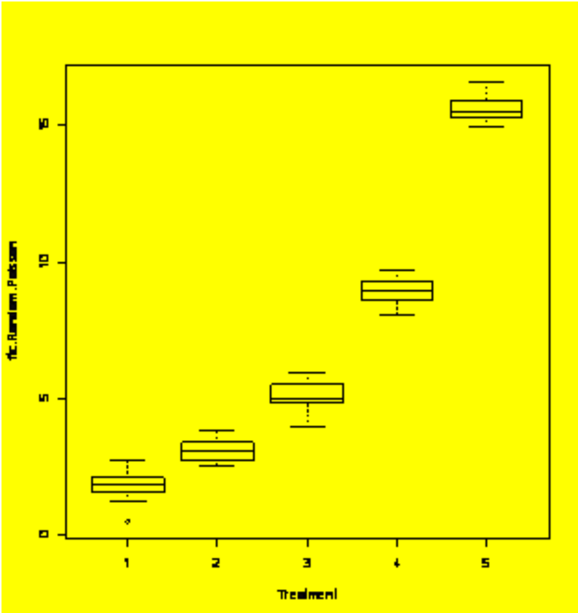

```
logY.aov=aov(log10(Y)~X)                                #Do the ANOVA
qqnorm(residuals(logY.aov)); qqline(residuals(logY.aov))    #Normal plot with reference line
```



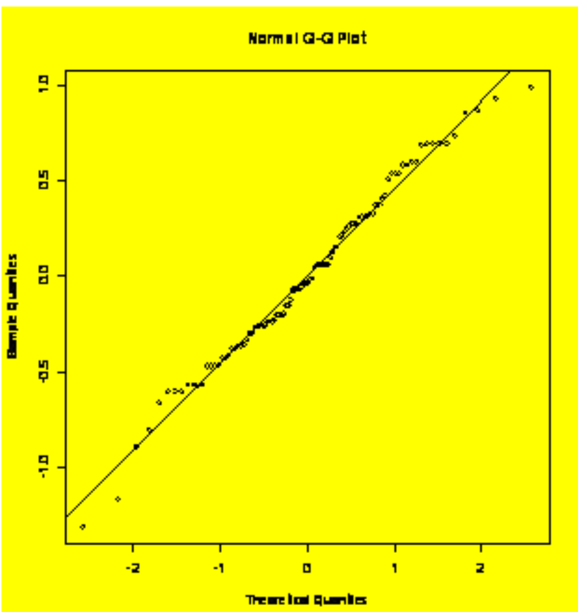

```
summary(logY.aov)                              #Reports the ANOVA of the log-transformed data

            Df Sum Sq Mean Sq F value    Pr(>F)
X            4 4.1015  1.0254  36.669 6.371e-15 ***
Residuals   55 1.5380  0.0280
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1


### Example 5.12 (p. 183) Plots of original and transformed Poisson random samples with different means.
Random.Poisson = c(rpois(20,3),rpois(20,9),rpois(20,27),rpois(20,81),rpois(20,243))        #Random Poisson samples
Treatment=gl(5,20,100)                                                                  #Treatment codes 1:5
plot(Random.Poisson~Treatment)                                                          #Plot of raw counts - trouble!
```




```
ftc.Random.Poisson=(sqrt(Random.Poisson)+sqrt(Random.Poisson+1))/2                      #Transform the counts
plot(ftc.Random.Poisson~Treatment)                                                      #Plot of the transformed counts
```



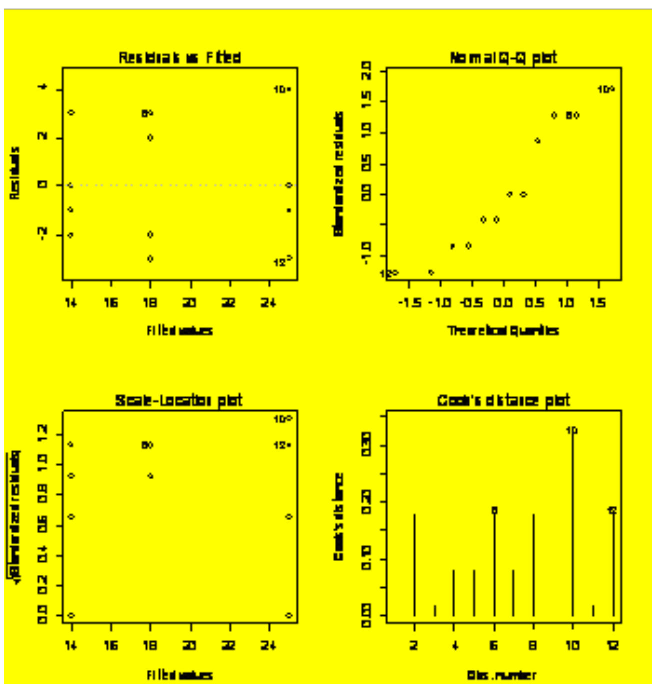

```
ftc.resid=residuals(aov(ftc.Random.Poisson~Treatment))              #ANOVA residuals after transform
qqnorm(ftc.resid);qqline(ftc.resid)                                 #Normal plot of residuals - looks great!
```




```
### Example 5.14 (p. 188) Sample-size and power calculations for one-way ANOVA.
### Note: The function power.anova.test specifies the problem in terms of the between-treatment
### and within-treatment variances, which have to be calculated first.
Biases=c(-5,5,0,0,0)                         #Treatment biases relative to grand mean
BTV=var(Biases)                              #Between treatment variance
#BTV=2*5^2/(5-1)                             #Alternative BTV calculation
WTV=4.2^2                                    #Within treatment variance
power.anova.test(groups=5,between.var=BTV,within.var=WTV,power=0.90)       #Find the sample size (Answer is n=7)

     Balanced one-way analysis of variance power calculation

         groups = 5
              n = 6.465695
    between.var = 12.5
     within.var = 17.64
      sig.level = 0.05
          power = 0.9

 NOTE: n is number in each group

power.anova.test(groups=5,n=7,between.var=BTV,within.var=WTV)              #Check the exact power for n=7

     Balanced one-way analysis of variance power calculation
```

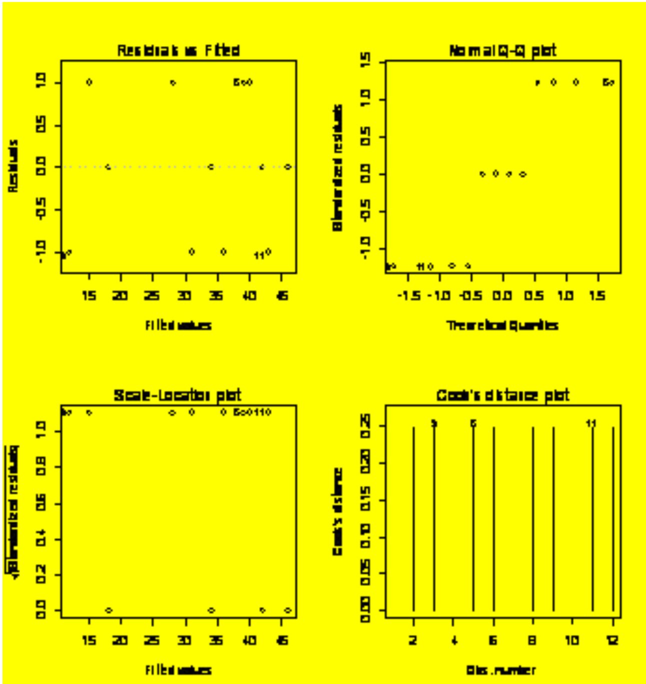
```
         groups = 5
              n = 7
    between.var = 12.5
     within.var = 17.64
      sig.level = 0.05
          power = 0.9279148

 NOTE: n is number in each group
```

```
##################################################################################

#              CHAPTER 6: Experiments for Multi-Way Classifications

##################################################################################

######################### WARNING!!! ############################################
### The analyses shown here using the aov() function are only valid for balanced
### experiment designs. If you have an unbalanced design or a balanced design with
### missing values, you MUST use the Anova() function in the car package with
### Type II sums of squares. (What R and SAS call Type II sums of squares are called
### Type III sums of squares in MINITAB and some other packages.)
##################################################################################

### Example 6.2 (p. 195) Review of one-way ANOVA.
Y=c(14,17,13,12,20,21,16,15,25,29,24,22)
Tr=gl(3,4,12)
Y.aov=aov(Y~Tr)
summary(Y.aov)

            Df  Sum Sq Mean Sq F value    Pr(>F)
Tr           2 248.000 124.000  16.909 0.0008949 ***
Residuals    9  66.000   7.333
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

par(mfrow=c(2,2))
plot(Y.aov)                                 #Residuals diagnostic plots
```

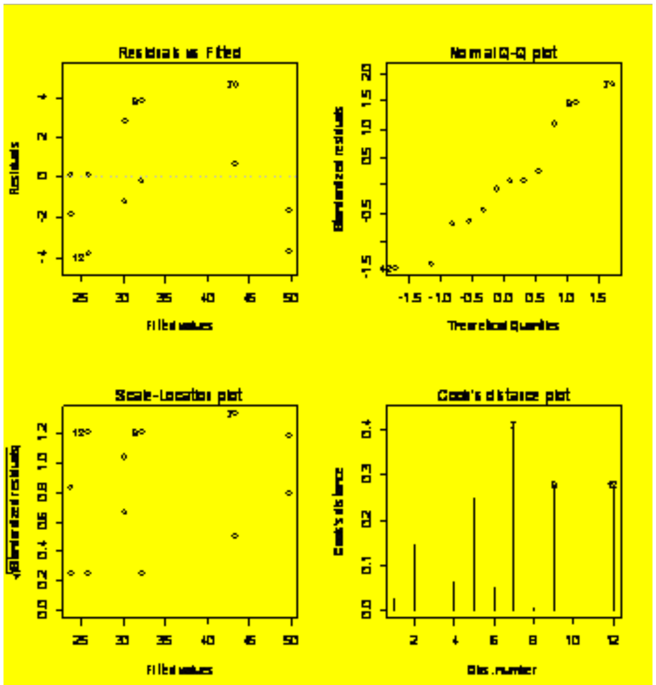



```
### Example 6.12 (p. 216) Two-way ANOVA.
Y=c(18,16,11,42,40,35,34,30,29,46,42,41)
A=gl(4,3,12)
B=gl(3,1,12)
Y.aov=aov(Y~A+B)
summary(Y.aov)

            Df  Sum Sq Mean Sq F value    Pr(>F)
A            3 1380.00  460.00     345 4.179e-07 ***
B            2   72.00   36.00      27     0.001 **
Residuals    6    8.00    1.33
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1
```

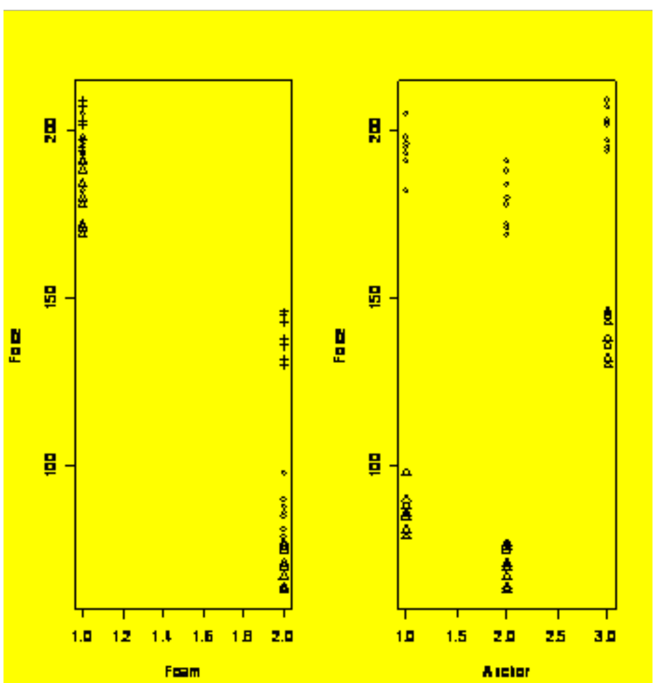
```
par(mfrow=c(2,2))
plot(Y.aov)
```




```
### Example 6.12 (p. 216) Two-way ANOVA with interaction.
Y=c(29,33,24,22,46,48,48,44,36,32,26,22)
A=gl(3,4,12)
B=gl(2,2,12)
Y.aov=aov(Y~A*B)
summary(Y.aov)

            Df Sum Sq Mean Sq F value    Pr(>F)
A            2 920.67  460.33 76.7222 5.329e-05 ***
B            1 120.33  120.33 20.0556   0.00420 **
A:B          2  44.67   22.33  3.7222   0.08888 .
Residuals    6  36.00    6.00
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

### The interaction term is not significant, so drop it from the model.
Y.aov=aov(Y~A+B)
summary(Y.aov)

            Df Sum Sq Mean Sq F value    Pr(>F)
A            2 920.67  460.33  45.653 4.212e-05 ***
B            1 120.33  120.33  11.934  0.008637 **
Residuals    8  80.67   10.08
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

par(mfrow=c(2,2))
plot(Y.aov)
```

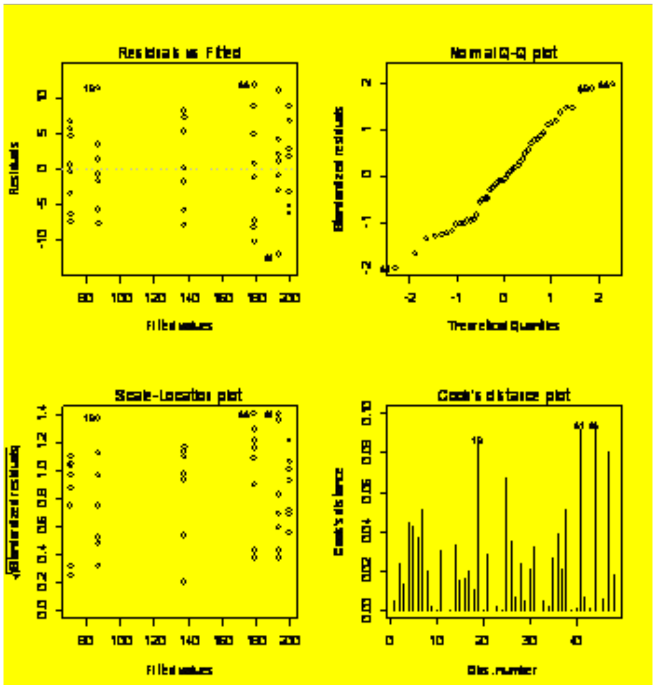



```
### Example 6.13 (p. 217) Analysis of a rotator cuff repair anchor.
Anchor=c(1,3,2,3,2,1,2,2,3,1,3,1,2,3,2,3,1,1,1,2,2,3,3,1,2,2,1,3,1,3,2,1,3,1,2,3,3,3,2,1,1,2,1,2,2,3,1,3)
Foam=c(1,1,2,2,1,2,1,2,2,1,1,2,2,2,1,1,2,1,2,1,2,2,1,1,1,2,2,1,1,2,1,2,1,1,2,2,2,1,1,2,1,2,2,1,2,1,1,2)
Force=c(191,194,75,146,171,79,188,76,136,195,207,86,71,145,184,195,81,198,98,178,77,138,202,193,169,63,90,
194,191,132,172,86,203,196,64,130,132,209,180,85,182,67,88,191,70,197,205,143)
par(mfrow=c(1,2))
plot(Force~Foam,pch=Anchor)
plot(Force~Anchor,pch=Foam)
```

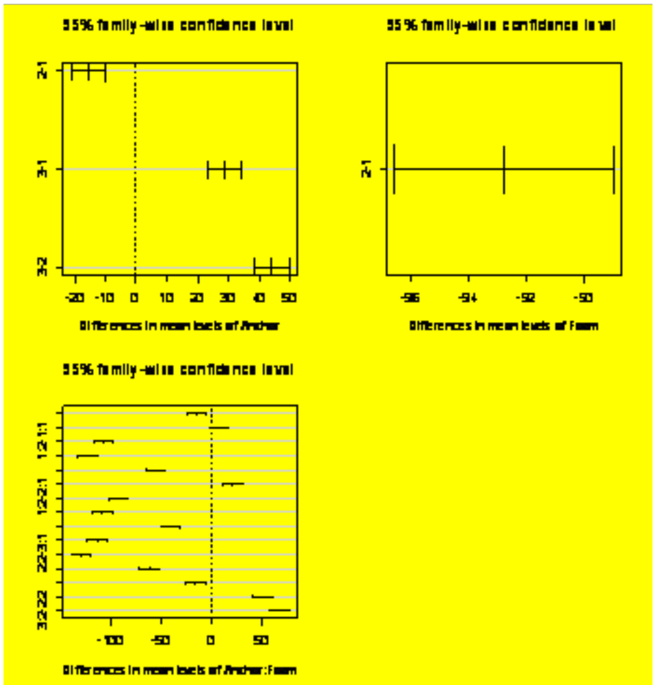



```
Anchor=factor(Anchor)
Foam=factor(Foam)
Force.aov=aov(Force~Anchor*Foam)
summary(Force.aov)

            Df Sum Sq Mean Sq  F value    Pr(>F)
Anchor       2  16084    8042  194.579 < 2.2e-16 ***
Foam         1 103324  103324 2499.943 < 2.2e-16 ***
Anchor:Foam  2   5556    2778   67.209 8.144e-14 ***
Residuals   42   1736      41
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1
```

```
par(mfrow=c(2,2))
plot(Force.aov)                                              #Residuals diagnostic plots
```

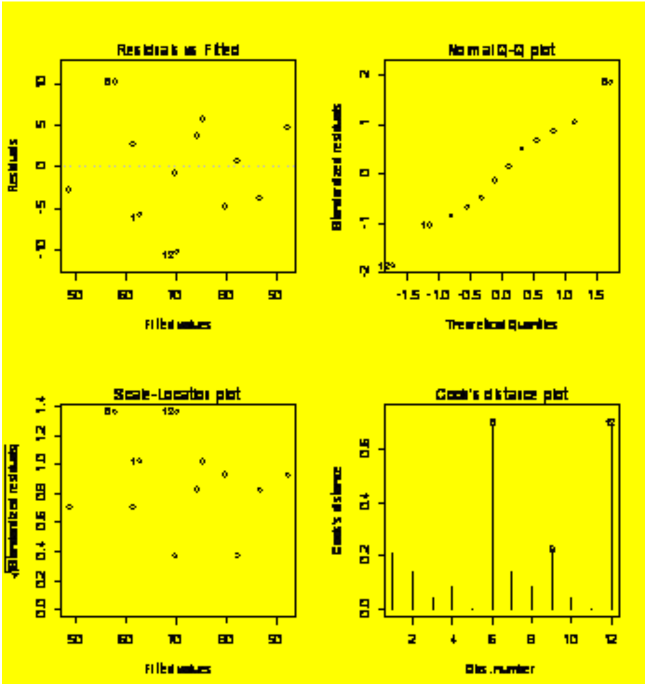



```
TukeyHSD(Force.aov)

  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = Force ~ Anchor * Foam)

$Anchor
        diff       lwr       upr
2-1 -15.5000 -21.02211 -9.977886
3-1  28.6875  23.16539 34.209614
3-2  44.1875  38.66539 49.709614

$Foam
         diff       lwr      upr
2-1 -92.79167 -96.53693 -89.0464

$"Anchor:Foam"
            diff         lwr         upr
2:1-1:1  -14.750  -24.345887   -5.154113
3:1-1:1    6.250   -3.345887   15.845887
1:2-1:1 -107.250 -116.845887  -97.654113
2:2-1:1 -123.500 -133.095887 -113.904113
3:2-1:1  -56.125  -65.720887  -46.529113
3:1-2:1   21.000   11.404113   30.595887
1:2-2:1  -92.500 -102.095887  -82.904113
2:2-2:1 -108.750 -118.345887  -99.154113
3:2-2:1  -41.375  -50.970887  -31.779113
1:2-3:1 -113.500 -123.095887 -103.904113
2:2-3:1 -129.750 -139.345887 -120.154113
3:2-3:1  -62.375  -71.970887  -52.779113
2:2-1:2  -16.250  -25.845887   -6.654113
3:2-1:2   51.125   41.529113   60.720887
3:2-2:2   67.375   57.779113   76.970887

par(mfrow=c(2,2))
plot(TukeyHSD(Force.aov))
```



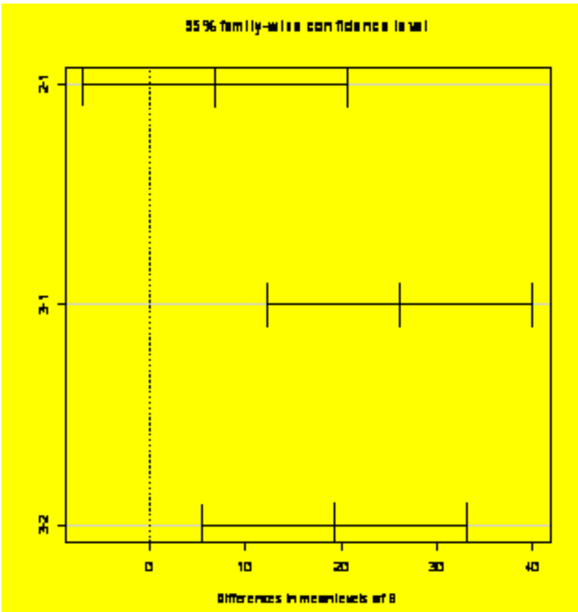

```
library(multcomp)                                     #Multiple comparisons package
summary(simint(Y~X, whichf="X",type = "Tukey"))       #TukeyHSD CIs and p values

        Simultaneous 95% confidence intervals: Tukey contrasts

Call:
simint.formula(formula = Y ~ X, type = "Tukey")

         Tukey contrasts for factor X

Contrast matrix:
        X1 X2 X3 X4 X5 X6 X7 X8 X9
X2-X1 0 -1  1  0  0  0  0  0  0  0
X3-X1 0 -1  0  1  0  0  0  0  0  0
X4-X1 0 -1  0  0  1  0  0  0  0  0
X5-X1 0 -1  0  0  0  1  0  0  0  0
X6-X1 0 -1  0  0  0  0  1  0  0  0
X7-X1 0 -1  0  0  0  0  0  1  0  0
X8-X1 0 -1  0  0  0  0  0  0  1  0
X9-X1 0 -1  0  0  0  0  0  0  0  1
X3-X2 0  0 -1  1  0  0  0  0  0  0
X4-X2 0  0 -1  0  1  0  0  0  0  0
X5-X2 0  0 -1  0  0  1  0  0  0  0
X6-X2 0  0 -1  0  0  0  1  0  0  0
X7-X2 0  0 -1  0  0  0  0  1  0  0
X8-X2 0  0 -1  0  0  0  0  0  1  0
X9-X2 0  0 -1  0  0  0  0  0  0  1
X4-X3 0  0  0 -1  1  0  0  0  0  0
X5-X3 0  0  0 -1  0  1  0  0  0  0
X6-X3 0  0  0 -1  0  0  1  0  0  0
X7-X3 0  0  0 -1  0  0  0  1  0  0
X8-X3 0  0  0 -1  0  0  0  0  1  0
X9-X3 0  0  0 -1  0  0  0  0  0  1
X5-X4 0  0  0  0 -1  1  0  0  0  0
X6-X4 0  0  0  0 -1  0  1  0  0  0
X7-X4 0  0  0  0 -1  0  0  1  0  0
X8-X4 0  0  0  0 -1  0  0  0  1  0
X9-X4 0  0  0  0 -1  0  0  0  0  1
X6-X5 0  0  0  0  0 -1  1  0  0  0
X7-X5 0  0  0  0  0 -1  0  1  0  0
X8-X5 0  0  0  0  0 -1  0  0  1  0
X9-X5 0  0  0  0  0 -1  0  0  0  1
X7-X6 0  0  0  0  0  0 -1  1  0  0
X8-X6 0  0  0  0  0  0 -1  0  1  0
X9-X6 0  0  0  0  0  0 -1  0  0  1
X8-X7 0  0  0  0  0  0  0 -1  1  0
X9-X7 0  0  0  0  0  0  0 -1  0  1
X9-X8 0  0  0  0  0  0  0  0 -1  1

Absolute Error Tolerance:  0.001

 95 % quantile:  3.198

Coefficients:
```

```
      Estimate  2.5 % 97.5 % t value Std.Err. p raw p Bonf p adj
X2-X1    1.956 -2.720  6.631   1.338    1.462 0.185  1.000 0.916
X3-X1    0.256 -4.420  4.931   0.175    1.462 0.862  1.000 1.000
X4-X1    1.944 -2.731  6.620   1.330    1.462 0.188  1.000 0.919
X5-X1    0.678 -3.998  5.353   0.464    1.462 0.644  1.000 1.000
X6-X1    0.967 -3.709  5.642   0.661    1.462 0.511  1.000 0.999
X7-X1    2.133 -2.542  6.809   1.459    1.462 0.149  1.000 0.870
X8-X1    3.700 -0.975  8.375   2.531    1.462 0.014  0.488 0.235
X9-X1    1.667 -3.009  6.342   1.140    1.462 0.258  1.000 0.966
X3-X2   -1.700 -6.375  2.975  -1.163    1.462 0.249  1.000 0.962
X4-X2   -0.011 -4.686  4.664  -0.008    1.462 0.994  1.000 1.000
X5-X2   -1.278 -5.953  3.398  -0.874    1.462 0.385  1.000 0.994
X6-X2   -0.989 -5.664  3.686  -0.676    1.462 0.501  1.000 0.999
X7-X2    0.178 -4.498  4.853   0.122    1.462 0.904  1.000 1.000
X8-X2    1.744 -2.931  6.420   1.193    1.462 0.237  1.000 0.955
X9-X2   -0.289 -4.964  4.386  -0.198    1.462 0.844  1.000 1.000
X4-X3    1.689 -2.986  6.364   1.155    1.462 0.252  1.000 0.963
X5-X3    0.422 -4.253  5.098   0.289    1.462 0.774  1.000 1.000
X6-X3    0.711 -3.964  5.386   0.486    1.462 0.628  1.000 1.000
X7-X3    1.878 -2.798  6.553   1.285    1.462 0.203  1.000 0.933
X8-X3    3.444 -1.231  8.120   2.356    1.462 0.021  0.763 0.324
X9-X3    1.411 -3.264  6.086   0.965    1.462 0.338  1.000 0.988
X5-X4   -1.267 -5.942  3.409  -0.867    1.462 0.389  1.000 0.994
X6-X4   -0.978 -5.653  3.698  -0.669    1.462 0.506  1.000 0.999
X7-X4    0.189 -4.486  4.864   0.129    1.462 0.898  1.000 1.000
X8-X4    1.756 -2.920  6.431   1.201    1.462 0.234  1.000 0.954
X9-X4   -0.278 -4.953  4.398  -0.190    1.462 0.850  1.000 1.000
X6-X5    0.289 -4.386  4.964   0.198    1.462 0.844  1.000 1.000
X7-X5    1.456 -3.220  6.131   0.996    1.462 0.323  1.000 0.985
X8-X5    3.022 -1.653  7.698   2.067    1.462 0.042  1.000 0.503
X9-X5    0.989 -3.686  5.664   0.676    1.462 0.501  1.000 0.999
X7-X6    1.167 -3.509  5.842   0.798    1.462 0.427  1.000 0.997
X8-X6    2.733 -1.942  7.409   1.870    1.462 0.066  1.000 0.637
X9-X6    0.700 -3.975  5.375   0.479    1.462 0.633  1.000 1.000
X8-X7    1.567 -3.109  6.242   1.072    1.462 0.287  1.000 0.976
X9-X7   -0.467 -5.142  4.209  -0.319    1.462 0.750  1.000 1.000
X9-X8   -2.033 -6.709  2.642  -1.391    1.462 0.169  1.000 0.898


### Example 6.14 (p. 224) Commands to create the 3x8x5 factorial design matrix with two replicates.
### Variables are going to be named A, B, C, and Rep:
A=gl(3,80,240)
B=gl(8,10,240)
C=gl(5,2,24)
Rep=gl(2,1,120)
expt.design = data.frame(A,B,C)
expt.design

    A B C
1   1 1 1
2   1 1 1
3   1 1 2
4   1 1 2
...
235 3 8 5
236 3 8 5
237 3 8 1
238 3 8 1
239 3 8 2
240 3 8 2

### Example 6.15 (p. 226) Analysis of a two-way design with blocking on replicates.
Y=c(57,75,46,78,69,68,97,83,81,64,83,60)
A=c(1,2,1,2,2,1,2,2,1,1,2,1)
B=c(1,1,3,3,2,2,1,3,1,3,2,2)
Block=gl(2,6,12)
A=factor(A)
B=factor(B)
Block=factor(Block)
Y.aov=aov(Y~Block+A*B)
summary(Y.aov)

            Df Sum Sq Mean Sq F value  Pr(>F)
Block        1 468.75  468.75  6.4081 0.05244 .
A            1 990.08  990.08 13.5350 0.01431 *
B            2 208.50  104.25  1.4252 0.32375
A:B          2  93.17   46.58  0.6368 0.56706
Residuals    5 365.75   73.15
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

par(mfrow=c(2,2))
plot(Y.aov)
```



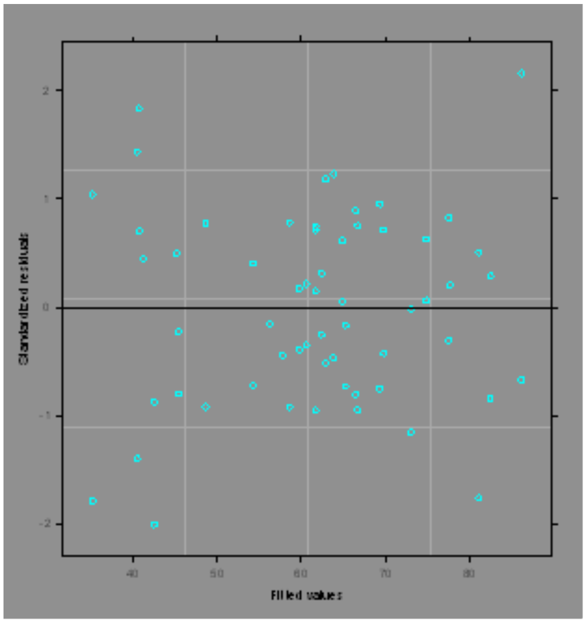

```
##################################################################################

#              CHAPTER 7: Advanced ANOVA Topics

##################################################################################

### Example 7.1 (p. 233) Analysis of a three-variable Latin Square design.
A=c(1,1,1,2,2,2,3,3,3,1,1,1,2,2,2,3,3,3)      #equivalent to A=gl(3,3,18)
B=c(1,2,3,1,2,3,1,2,3,1,2,3,1,2,3,1,2,3)      #equivalent to B=gl(3,1,18)
C=c(1,2,3,2,3,1,3,1,2,1,2,3,2,3,1,3,1,2)
Y=c(63,73,78,66,63,92,59,49,99,52,67,82,60,62,73,46,73,79)
A=factor(A)
B=factor(B)
C=factor(C)
Y.lm = lm(Y~A+B+C)
anova(Y.lm)                                   #Report the ANOVA table ...

Analysis of Variance Table

Response: Y
          Df  Sum Sq Mean Sq F value    Pr(>F)
A          2   12.33    6.17  0.0782 0.9252806
B          2 2210.33 1105.17 14.0163 0.0009436 ***
C          2  268.00  134.00  1.6995 0.2274283
Residuals 11  867.33   78.85
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

summary(Y.lm)                                 #And summary statistics including coefficients.

Call:
lm(formula = Y ~ A + B + C)

Residuals:
     Min       1Q   Median       3Q      Max
-12.6667  -4.2917   0.9167   5.2917  11.3333

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  56.5000     5.5374  10.203 6.04e-07 ***
A2            0.1667     5.1267   0.033 0.974648
A3           -1.6667     5.1267  -0.325 0.751207
B2            6.8333     5.1267   1.333 0.209515
B3           26.1667     5.1267   5.104 0.000342 ***
C2            7.0000     5.1267   1.365 0.199397
C3           -2.0000     5.1267  -0.390 0.703899
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Residual standard error: 8.88 on 11 degrees of freedom
Multiple R-Squared: 0.7417,     Adjusted R-squared: 0.6008
F-statistic: 5.265 on 6 and 11 DF,  p-value: 0.008713
```

```
TukeyHSD(aov(Y~A+B+C),which="B")             #Reports Tukey HSD CIs for differences between the means of B.

  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = Y ~ A + B + C)

$B
         diff       lwr      upr
2-1  6.833333 -7.01309 20.67976
3-1 26.166667 12.32024 40.01309
3-2 19.333333  5.48691 33.17976

plot(TukeyHSD(aov(Y~A+B+C),which="B"))       #Plots the CIS for B
```

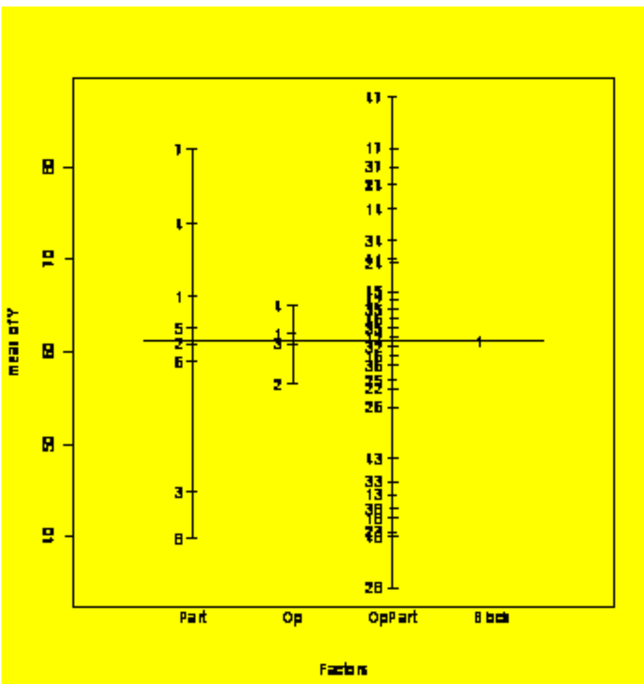



```
library(multcomp)
summary(simint(Y~A+B+C,whichf="B",type="Tukey"))#TukeyHSD CIs and p values

        Simultaneous 95% confidence intervals: Tukey contrasts

Call:
simint.formula(formula = Y ~ A + B + C, whichf = "B", type = "Tukey")

         Tukey contrasts for factor B, covariables:  A + C

Contrast matrix:
            B1 B2 B3
B2-B1 0 0 0 -1  1  0 0 0
B3-B1 0 0 0 -1  0  1 0 0
B3-B2 0 0 0  0 -1  1 0 0

Absolute Error Tolerance:  0.001

 95 % quantile:  2.701

Coefficients:
      Estimate  2.5 % 97.5 % t value Std.Err. p raw p Bonf p adj
B2-B1    6.833 -7.011 20.678   1.333    5.127 0.210  0.629 0.407
B3-B1   26.167 12.322 40.011   5.104    5.127 0.000  0.001 0.001
B3-B2   19.333  5.489 33.178   3.771    5.127 0.003  0.009 0.008


### Example 7.5 (p. 242) GR&R study analysis using random effects model.
### Warning: The R code required to perform this variance components analysis is cryptic!
### For help on the methods from Chapter 7, see Pinheiro and Bates, Mixed-Effects Models in S and S-Plus, Springer,
2000.

Y=c(65,68,60,63,44,45,75,76,63,66,59,60,81,83,42,42,62,63,56,56,38,43,68,71,57,57,55,53,79,77,32,
37,64,65,60,61,46,46,73,71,63,62,57,60,78,82,44,42,71,69,65,66,50,47,78,78,65,68,65,62,90,85,39,41)
Part=gl(8,2,64)                                          #Integers 1 to 8, two times in succession, 64 values total
Op=gl(4,16,64)
Y.aov=aov(Y~1+Error(Part*Op))                            #Part and Op are crossed random factors
summary(Y.aov)

Error: Part
          Df  Sum Sq Mean Sq F value Pr(>F)                 #Note: R refuses to report F and p values for random
```

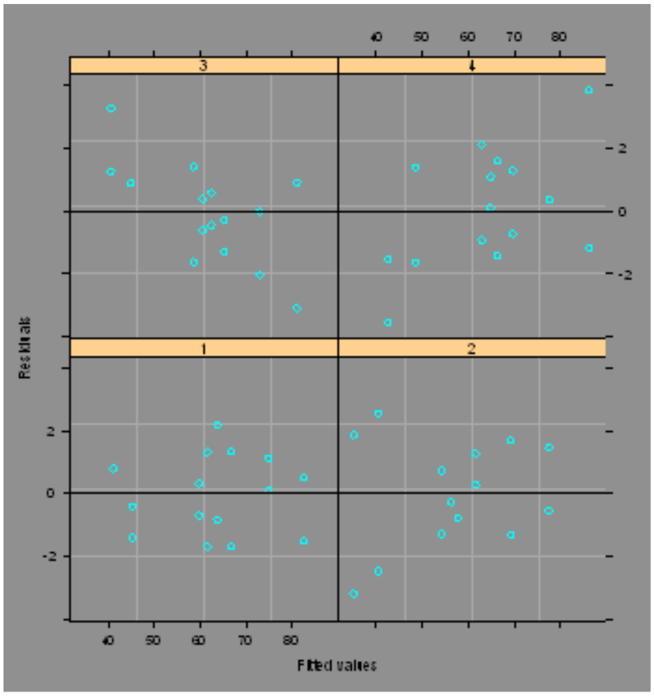
```
effects
Residuals  7 10684.0  1526.3

Error: Op
          Df Sum Sq Mean Sq F value Pr(>F)
Residuals  3 587.92  195.97

Error: Part:Op
          Df Sum Sq Mean Sq F value Pr(>F)
Residuals 21 95.203   4.533

Error: Within
          Df Sum Sq Mean Sq F value Pr(>F)
Residuals 32 99.500   3.109

### Now use lme() to extract the variance components:
OpPart = 10*as.numeric(Op)+as.numeric(Part)                #Create a code for the Op*Part interaction
Block=rep(1,64)                                            #All of the observations come from a single
block
Part=factor(Part)                                          #Change variables from quantitative to
qualitative
Op=factor(Op)
OpPart = factor(OpPart)

grr.dataframe=data.frame(Y,Part,Op,OpPart,Block)
library(nlme)                                              #non-linear mixed effects package
grr.groupedData = groupedData(Y~1|Block,data=grr.dataframe) #This object wraps the dataframe and its
equation
grr.lme=lme(Y~1,data=grr.groupedData,random=pdBlocked(list(pdIdent(~Part-1),pdIdent(~Op-
1),pdIdent(~OpPart-1))))              #???

VarCorr(grr.lme)                                           #Calculate the variance components

Block = pdIdent(Part - 1), pdIdent(Op - 1), pdIdent(OpPart - 1)
         Variance    StdDev
Part1    190.2137137 13.7917988
Op1       11.9641758  3.4589270
OpPart11   0.7119273  0.8437578
Residual   3.1094800  1.7633718

intervals(grr.lme)                                  #Calculates confidence intervals for the variance
components

Approximate 95% confidence intervals

 Fixed effects:
              lower     est.    upper
(Intercept) 50.72553 61.07813 71.43072
attr(,"label")
[1] "Fixed effects:"

 Random Effects:
  Level: Block
                   lower       est.    upper
sd(Part - 1)   8.1555365 13.7917988 23.32326
sd(Op - 1)     1.5247709  3.4589270  7.84654
sd(OpPart - 1) 0.2833372  0.8437578  2.51265

 Within-group standard error:
   lower     est.    upper
1.380988 1.763372 2.251634

plot(grr.lme)                                       #The default plot is residuals vs. predicted
values.
```



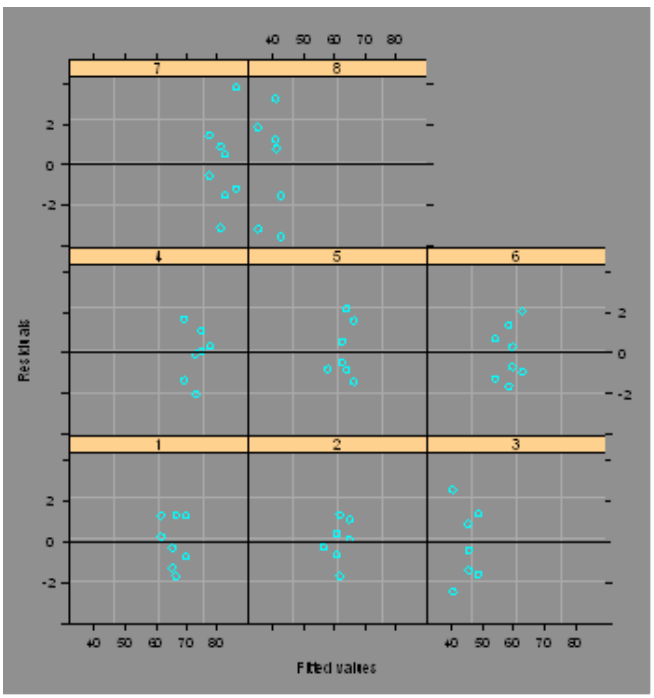

```
plot.design(grr.groupedData)                        #Main effects plot for the groupedData object
```



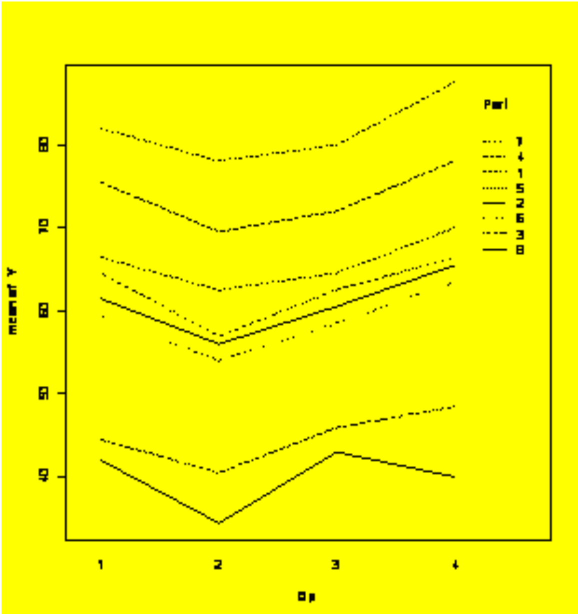

```
plot(grr.lme,form=resid(.)~fitted(.)|Op,abline=0)     #Plots residuals vs. fitted values by Op with 0 reference line.
```



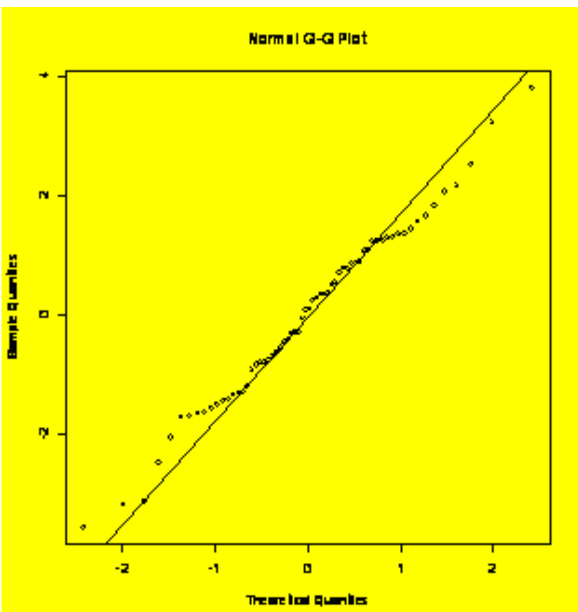

```
plot(grr.lme,form=resid(.)~fitted(.)|Part,abline=0)   #Plots residuals vs. fitted values by Part ...
```




```
interaction.plot(Op,Part,Y)                            #Interaction plot
```

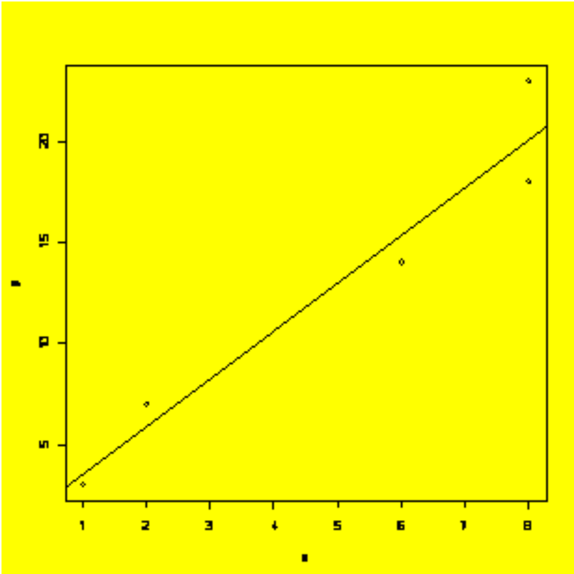



```
qqnorm(resid(grr.lme)); qqline(resid(grr.lme))                        #Residuals normal plot
```

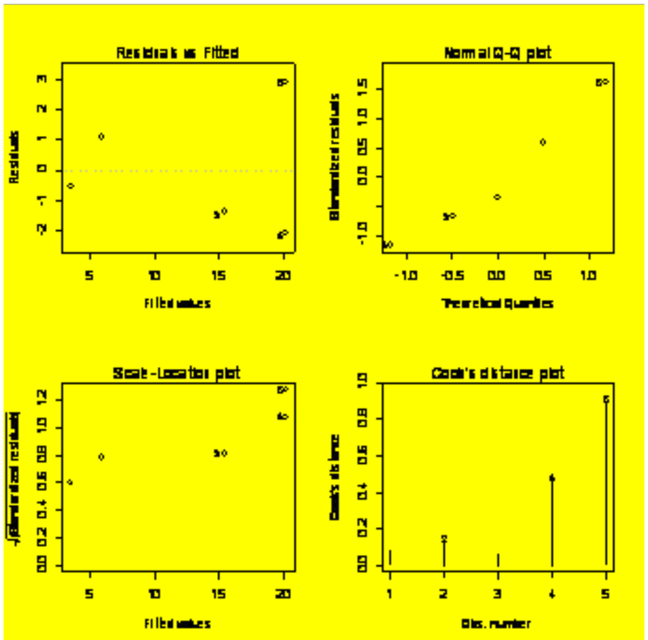



```
### Example 7.7 (p. 249) Analysis of a nested design.
Batch=c(1,1,1,1,1,1,1,1,2,2,2,2,2,2,2,2,3,3,3,3,3,3,3,3,1,1,1,1,1,1,1,1,2,2,2,2,2,2,2,2,3,3,3,3,3,3,3,3)
#or use Batch=gl(3,8,48)
Tote=c(1,1,2,2,3,3,4,4,1,1,2,2,3,3,4,4,1,1,2,2,3,3,4,4,1,1,2,2,3,3,4,4,1,1,2,2,3,3,4,4,1,1,2,2,3,3,4,4)
#or use Tote=gl(4,2,48)
Cup=c(1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2)
#or use Cup=gl(2,1,48)
Y=c(12.8,12.8,13,12.9,13.5,13.5,12.9,13.2,11.2,11.2,12.3,12.3,10.7,10.4,11.8,12.1,11.5,11.5,11.3,11.3,
11.6,11.4,11.2,11.1,12.5,12.2,13,12.8,13.5,13.4,12.5,12.7,11.3,11,12.4,12,10.5,10.9,11.5,11.8,11.7,11.3,
11.5,11.4,11.7,11.2,11,11.2)
Batch=factor(Batch)
Tote = factor(Tote)
Cup=factor(Cup)
Y.aov=aov(Y~1+Error(Batch/Tote/Cup))                                  #Fully nested random factors
summary(Y.aov)

Error: Batch
          Df Sum Sq Mean Sq F value Pr(>F)
Residuals  2 25.183  12.592

Error: Batch:Tote
          Df Sum Sq Mean Sq F value Pr(>F)
Residuals  9 8.1544  0.9060

Error: Batch:Tote:Cup
```

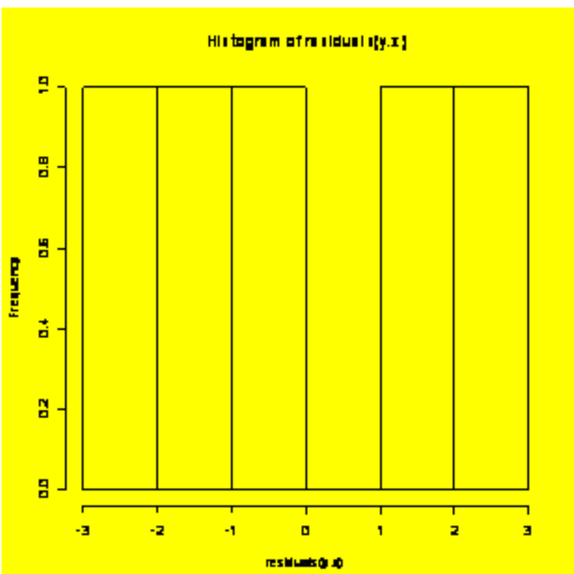
```
          Df  Sum Sq Mean Sq F value Pr(>F)
Residuals 12 0.43250 0.03604

Error: Within
          Df  Sum Sq Mean Sq F value Pr(>F)
Residuals 24 0.86500 0.03604

### Now use lme() to extract the variance components:
Block=rep(1,48)
Homogeneity.dataframe=data.frame(Batch,Tote,Cup,Y,Block)
library(nlme)
Homogeneity.groupedData=groupedData(Y~1|Block,Homogeneity.dataframe)
Homogeneity.lme=lme(Y~1,data=Homogeneity.groupedData,random=~1|Batch/Tote/Cup)
VarCorr(Homogeneity.lme)

            Variance     StdDev
Batch =     pdLogChol(1)
(Intercept) 0.7297169665 0.85423473
Tote =      pdLogChol(1)
(Intercept) 0.2176946855 0.46657763
Cup =       pdLogChol(1)
(Intercept) 0.0004155194 0.02038429
Residual    0.0357570961 0.18909547


### Example 7.8 (p. 254) Power calculation for a 3x5 fixed-effects factorial design.
Falpha = qf(0.95,2,30)
Power = pf(Falpha,2,30,ncp=20.8)
Power                                                 #Display the result

[1] 0.02079381

### Alternative solution combines two steps.
Power = 1-pf(qf(0.95,2,30),2,30,ncp=20.8)
Power

[1] 0.9792062

### Example 7.9 (p. 255) Another power calculation for the 3x5 factorial design.
Power = 1-pf(qf(0.95,4,30),4,30,ncp=12.5)
Power

[1] 0.7484825

### Example 7.10 (p. 256) Power calculation for a random effect.
Falpha = qf(0.95,7,32)
E.FA = 1+5*(30/50)^2
Power = pf(E.FA/Falpha,32,7)
Power

[1] 0.5733428

<>### Example 7.11 (p. 258) Power calculation for a fixed effect in a mixed model.
Power = 1-pf(qf(0.95,2,6),2,6,ncp=4*3/2*(1/0.8)^2)
Power


[1] 0.551472


#################################################################################

#              CHAPTER 8: Linear Regression

#################################################################################

### Example 8.14 (p. 299) Linear regression analysis.
### Note: There are only five points in this data set, so some of the graphs are pretty pointless
### but the methods shown are still valid.
x=c(1,2,6,8,8)
y=c(3,7,14,18,23)
y.x=lm(y~x)                                           #Fits y as a linear function of x
plot(x,y);abline(y.x)                                 #Creates the scatter plot with fitted line
```



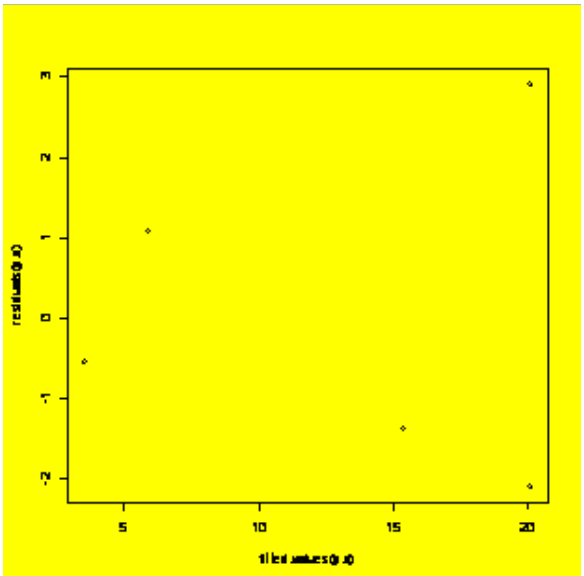
```
summary(y.x)                                          #Complete summary of the model

Call:
lm(formula = y ~ x)

Residuals:
      1       2       3       4       5
-0.5455  1.0909 -1.3636 -2.0909  2.9091

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   1.1818     2.0356   0.581  0.60226
x             2.3636     0.3501   6.751  0.00664 **
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Residual standard error: 2.322 on 3 degrees of freedom
Multiple R-Squared: 0.9382,     Adjusted R-squared: 0.9176
F-statistic: 45.57 on 1 and 3 DF,  p-value: 0.006639

anova(y.x)                                            #ANOVA table

Analysis of Variance Table

Response: y
          Df  Sum Sq Mean Sq F value   Pr(>F)
x          1 245.818 245.818  45.573 0.006639 **
Residuals  3  16.182   5.394
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

### The following functions provide access to other output from the model:
fitted.values(y.x)                                    #Vector of fitted values (y-hat)

        1         2         3         4         5
 3.545455  5.909091 15.363636 20.090909 20.090909

residuals(y.x)                                        #Vector of residuals

         1          2          3          4          5
-0.5454545  1.0909091 -1.3636364 -2.0909091  2.9090909

coefficients(y.x)                                     #Vector of regression coefficients

(Intercept)           x
   1.181818    2.363636

### Resdiduals diagnostic plots:
par(mfrow=c(2,2))
plot.lm(y.x)                                          #Creates four diagnostic plots
```



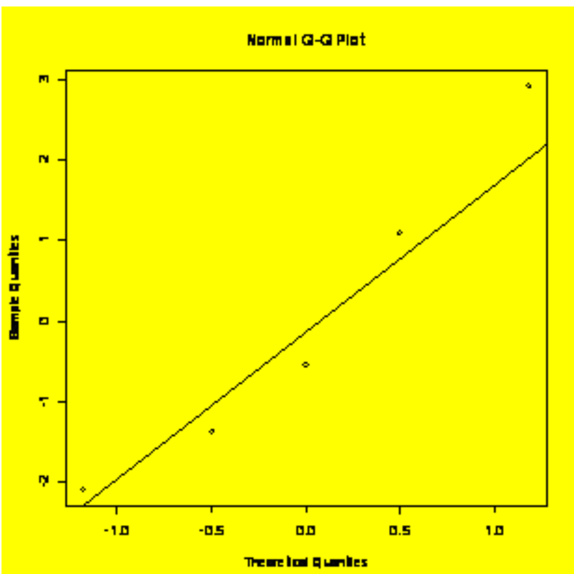

```
par(mfrow=c(1,1))
hist(residuals(y.x))                        #Creates histogram of the residuals
```



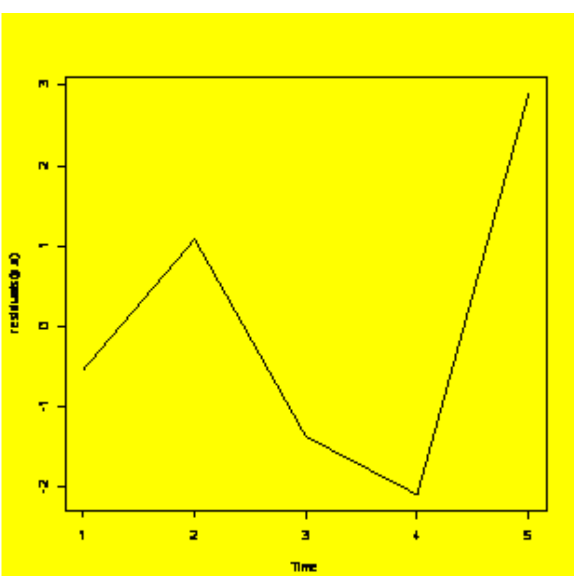

```
plot(fitted.values(y.x),residuals(y.x))     #Plot of residuals (y-axis) vs. fitted values (x axis)
```



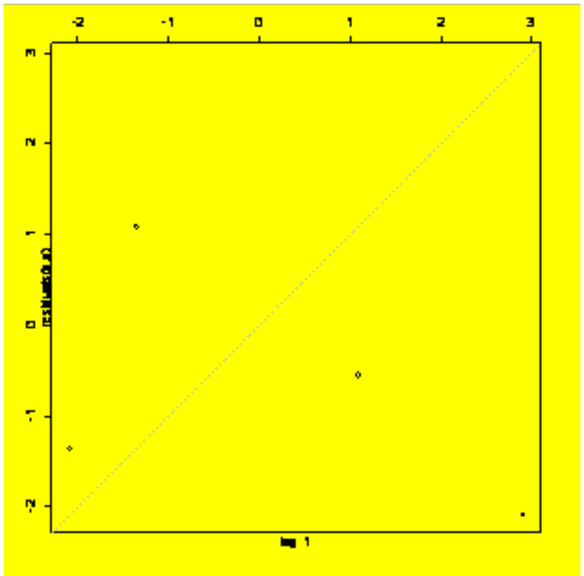

```
qqnorm(residuals(y.x)); qqline(residuals(y.x))       #Residuals normal plot
```



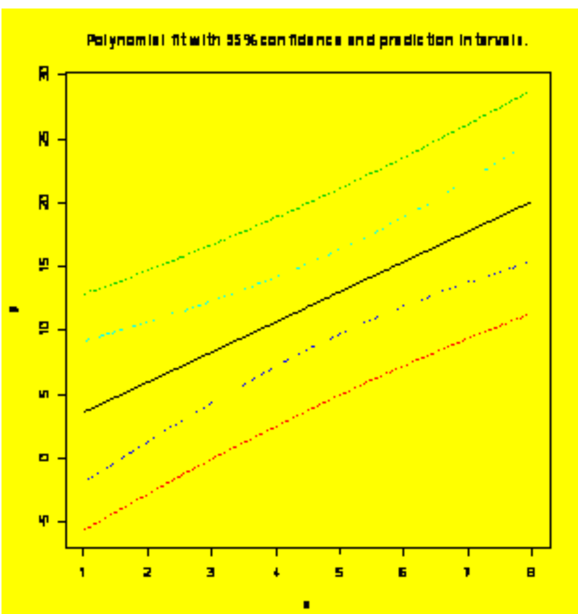

```
plot.ts(residuals(y.x))                              #Residuals run chart (i.e. time series)
```



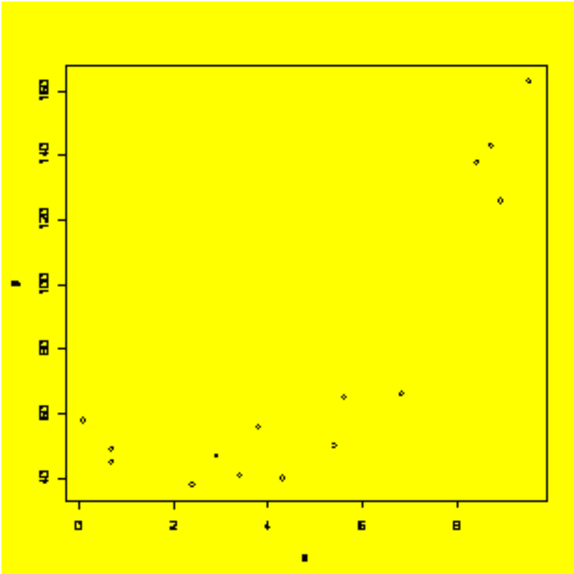

```
lag.plot(residuals(y.x),lags=1,do.lines=F,labels=F)  #Lag-1 residuals plot
```



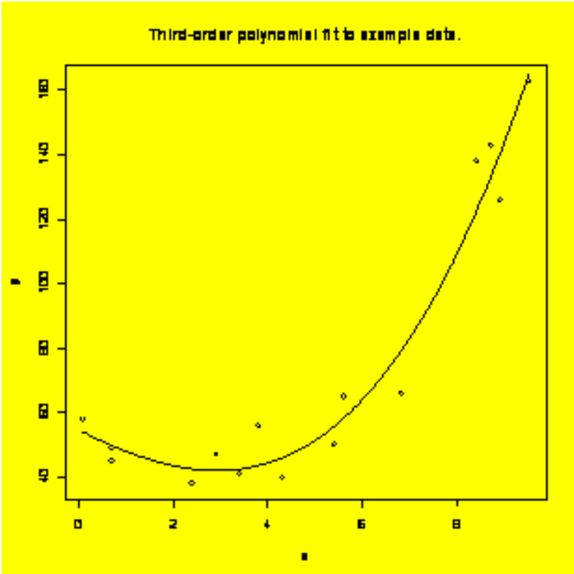
```
### Examples 8.7 (p. 289) and 8.10 (p. 292) Adding confidence and prediction intervals to the graph.
newx=data.frame(x=seq(min(x),max(x),diff(range(x))/100))     #Vector of new x values for plotting
newy.PL=predict(y.x,newdata=newx,interval="predict")         #Find prediction limits for the new x values
newy.CL=predict(y.x,newdata=newx,interval="confidence")      #Find confidence limits for the new x values
matplot(newx$x,cbind(newy.PL,newy.CL[,-1]),lty=c(1,2,2,3,3),type="l", xlab="x",ylab="y")        #Matrix plot
title("Polynomial fit with 95% confidence and prediction intervals.")
```



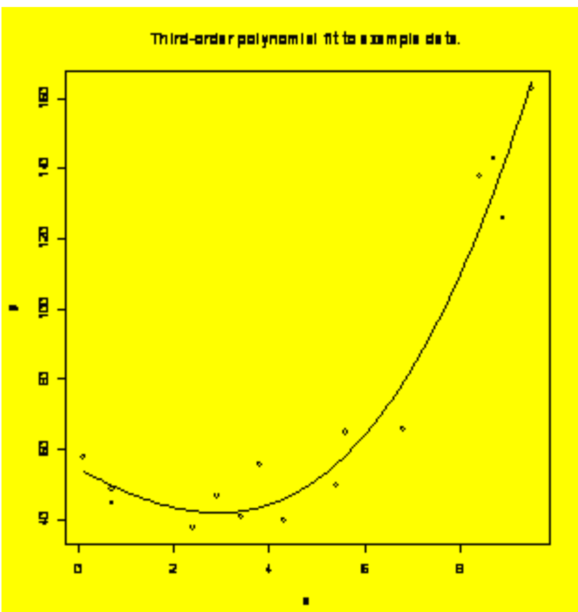
```
### Example 8.21 (p. 307) Fitting a third-order polynomial.
x=c(8.9,8.7,0.1,5.4,4.3,2.4,3.4,6.8,2.9,5.6,8.4,0.7,3.8,9.5,0.7)
y=c(126,143,58,50,40,38,41,66,47,65,138,49,56,163,45)
y.x=lm(y~x+I(x^2)+I(x^3))                                   #The I() notation is required to identify the powers of x
plot(x,y)                                                   #Scatter plot
```



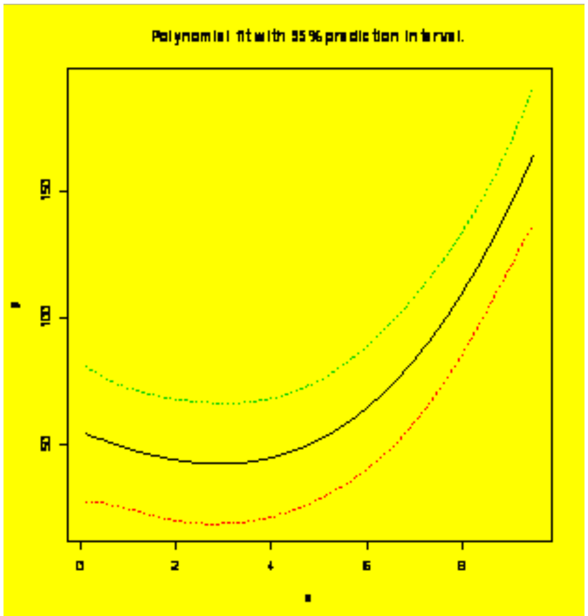
```
summary(y.x)                                                        #Complete summary of the model

Call:
lm(formula = y ~ x + I(x^2) + I(x^3))

Residuals:
    Min      1Q  Median      3Q     Max 
-14.549  -5.385  -1.476   5.787  15.397 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  54.7826     7.9324   6.906 2.57e-05 ***
x            -7.4856     8.1378  -0.920    0.377    
I(x^2)        0.6507     2.1505   0.303    0.768    
I(x^3)        0.1431     0.1513   0.945    0.365    
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1 

Residual standard error: 9.884 on 11 degrees of freedom
Multiple R-Squared: 0.9595,     Adjusted R-squared: 0.9484 
F-statistic: 86.76 on 3 and 11 DF,  p-value: 6.121e-08 

anova(y.x)                                                          #ANOVA table of the model

Analysis of Variance Table

Response: y
          Df  Sum Sq Mean Sq  F value    Pr(>F)    
x          1 18452.9 18452.9 188.8771 2.852e-08 ***
I(x^2)     1  6889.2  6889.2  70.5152 4.108e-06 ***
I(x^3)     1    87.3    87.3   0.8935    0.3648    
Residuals 11  1074.7    97.7                       
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1 


### Example 8.21, Figure 8.14 (p. 308) Create the scatter plot with the superimposed fitted function.
### First (brute force) method:
x.forplot=seq(min(x),max(x),diff(range(x))/100)                     #Vector of x for plotting
b=coefficients(y.x)                                                 #Cubic equation coefficients
y.forplot=b[1]+b[2]*x.forplot+b[3]*x.forplot^2+b[4]*x.forplot^3     #Vector of fitted y for plotting
plot(x,y,pch=1)                                                     #Make the scatter plot
lines(x.forplot,y.forplot,type="l")                                 #Add the fitted curve
title("Third-order polynomial fit to example data.")                #Add the title
```



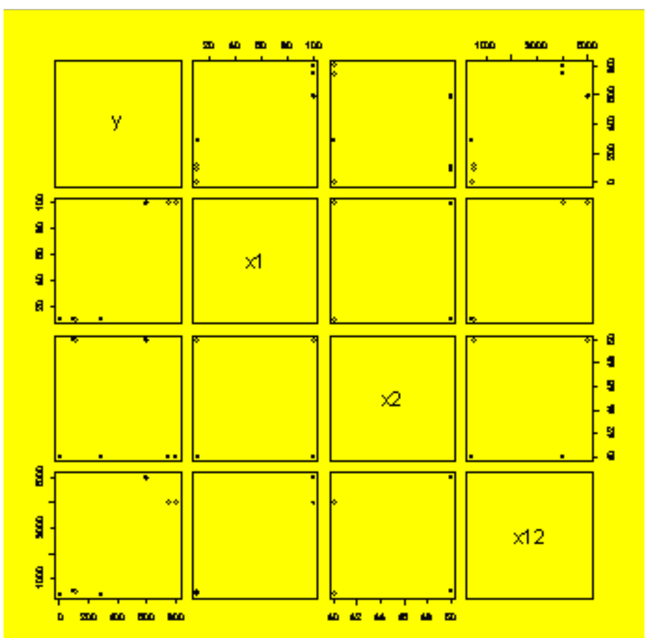

```
### Example 8.21, Figure 8.14 (p. 308) Create the scatter plot with the superimposed fitted function.
### Second method using predict():
newx=data.frame(x=seq(min(x),max(x),diff(range(x))/100))             #Vector of new x values for plotting
newy=predict(y.x,newdata=newx)                                       #Find y-hat for the new x values
plot(x,y);lines(newx$x,newy);title("Third-order polynomial fit to example data.")
```




```
### Example 8.21, Extra: Polynomial fit with 95% prediction interval.
newy=predict(y.x,newdata=newx,interval="predict")                    #Find the fit and prediction limits
matplot(newx$x,newy,lty=c(1,2,2),type="l", xlab = "x", ylab="y")
title("Polynomial fit with 95% prediction interval.")
```

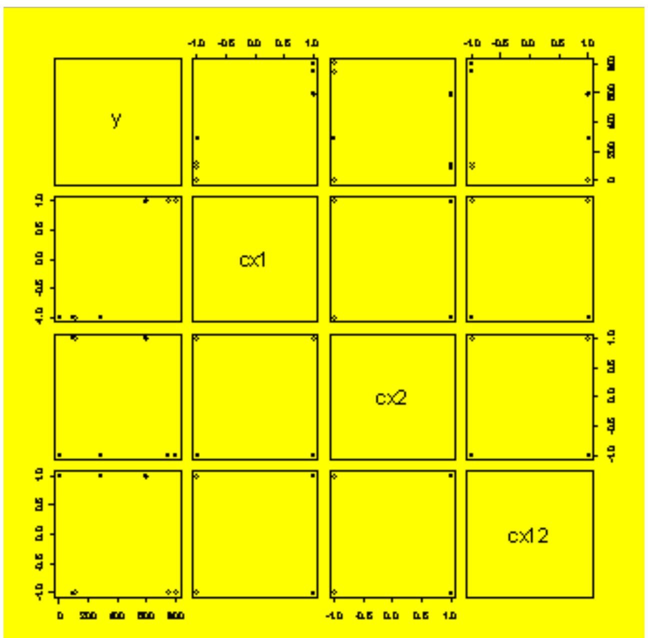



```
### Example 8.27, Figure 8.26 (p. 323) Matrix plot of response and uncoded predictors.
x1=c(10,10,10,10,100,100,100,100)
x2=c(40,40,50,50,40,40,50,50)
y=c(286,1,114,91,803,749,591,598)
x12=x1*x2
y.x1x2=data.frame(y,x1,x2,x12)
library(lattice)
pairs(y.x1x2)
```



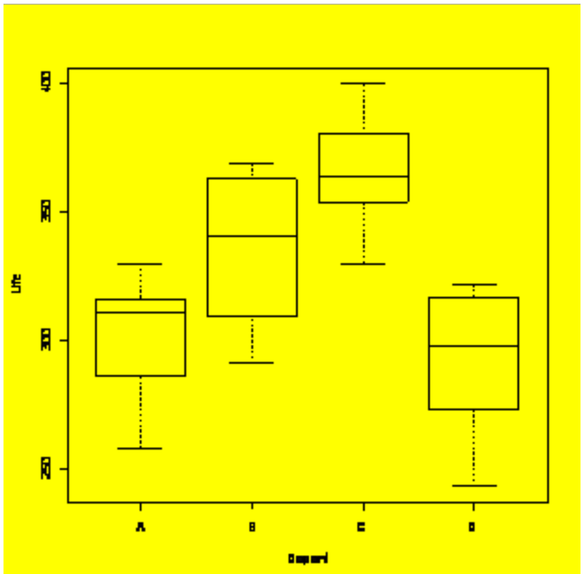

```
### Example 8.27, Figure 8.27 (p. 324) Multiple regression of y=f(x1,x2,x12) using uncoded variables.
MR.Uncoded=lm(y~x1+x2+x12)
summary(MR.Uncoded)

Call:
lm(formula = y ~ x1 + x2 + x12)

Residuals:
     1      2      3      4      5      6      7      8
 142.5 -142.5   11.5  -11.5   27.0  -27.0   -3.5    3.5

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 174.7778   520.2841   0.336    0.754
x1           13.2722     7.3214   1.813    0.144
x2           -2.5389    11.4912  -0.221    0.836
```

```
x12          -0.1561     0.1617  -0.965    0.389

Residual standard error: 102.9 on 4 degrees of freedom
Multiple R-Squared: 0.9403,     Adjusted R-squared: 0.8955
F-statistic: 20.99 on 3 and 4 DF,  p-value: 0.006554

anova(MR.Uncoded)

Analysis of Variance Table

Response: y
          Df Sum Sq Mean Sq F value   Pr(>F)
x1         1 632250  632250 59.7033 0.001511 **
x2         1  24753   24753  2.3374 0.201027
x12        1   9870    9870  0.9320 0.389005
Residuals  4  42360   10590
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1


### Example 8.27, Figure 8.28 (p. 325) Matrix plot of response and coded predictors.
cx1=(x1-55)/45                                              #Code the levels of x1 and x2
cx2=(x2-45)/5
cx12=cx1*cx2
y.cx1cx2=data.frame(y,cx1,cx2,cx12)
pairs(y.cx1cx2)                                             #Matrix plot of y, cx1, cx2, and cx12
```

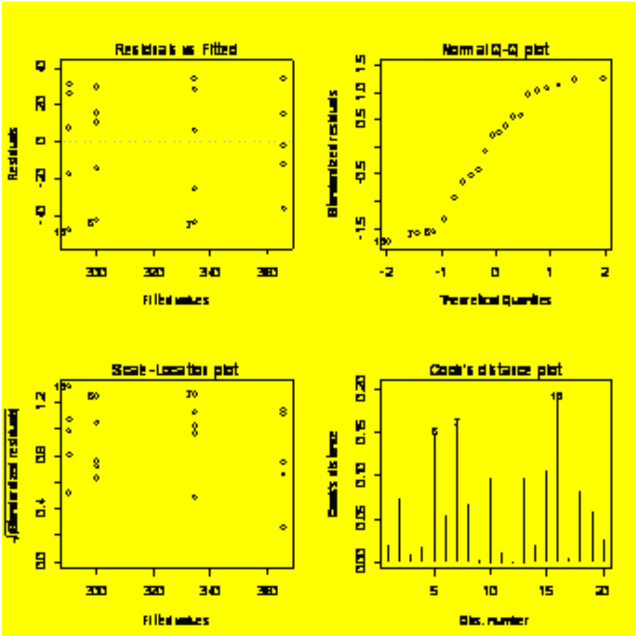



```
### Example 8.27, Figure 8.29 (p. 326) Multiple regression of y=f(cx1,cx2,cx12) using coded variables.
MR.Coded=lm(y~cx1+cx2+cx12)
summary(MR.Coded)

Call:
lm(formula = y ~ cx1 + cx2 + cx12)

Residuals:
     1      2      3      4      5      6      7      8
 142.5 -142.5   11.5  -11.5   27.0  -27.0   -3.5    3.5

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   404.12      36.38  11.107 0.000374 ***
cx1           281.13      36.38   7.727 0.001511 **
cx2           -55.62      36.38  -1.529 0.201027
cx12          -35.13      36.38  -0.965 0.389005
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Residual standard error: 102.9 on 4 degrees of freedom
Multiple R-Squared: 0.9403,     Adjusted R-squared: 0.8955
F-statistic: 20.99 on 3 and 4 DF,  p-value: 0.006554

anova(MR.Coded)

Analysis of Variance Table

Response: y
```

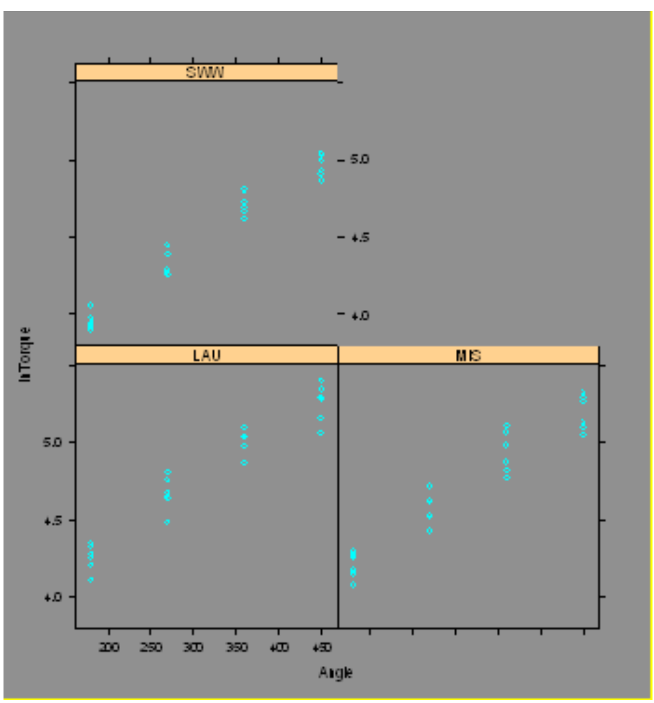
```
          Df Sum Sq Mean Sq F value   Pr(>F)
cx1        1 632250  632250 59.7033 0.001511 **
cx2        1  24753   24753  2.3374 0.201027
cx12       1   9870    9870  0.9320 0.389005
Residuals  4  42359   10590
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

### Example 8.29, Figure 8.30 (p. 328) One-way ANOVA of Life=f(Dopant).
Life=c(316,330,311,286,258,309,291,363,341,369,354,364,400,381,330,243,298,322,317,273)
Dopant=c("A","A","A","A","A","B","B","B","B","B","C","C","C","C","C","D","D","D","D","D")
Dopant=factor(Dopant)
plot(Life~Dopant)
```



```
Life.Dopant=lm(Life~Dopant)
summary(Life.Dopant)

Call:
lm(formula = Life ~ Dopant)

Residuals:
   Min     1Q Median     3Q    Max
 -47.6  -19.6    6.9   26.9   34.4

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   300.20      13.68  21.951 2.27e-13 ***
DopantB        34.40      19.34   1.779  0.09430 .
DopantC        65.60      19.34   3.392  0.00372 **
DopantD        -9.60      19.34  -0.496  0.62638
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Residual standard error: 30.58 on 16 degrees of freedom
Multiple R-Squared: 0.5416,     Adjusted R-squared: 0.4557
F-statistic: 6.302 on 3 and 16 DF,  p-value: 0.005005

### Note for above: R uses the first treatment group as the reference level, so its coefficient is zero by definition.
### This convention is different from some other programs where the reference level is the mean of all levels.

anova(Life.Dopant)

Analysis of Variance Table

Response: Life
          Df  Sum Sq Mean Sq F value   Pr(>F)
Dopant     3 17679.2  5893.1  6.3019 0.005005 **
Residuals 16 14962.0   935.1
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

par(mfrow=c(2,2))
plot(Life.Dopant)                                           #Residuals diagnostic plots
```

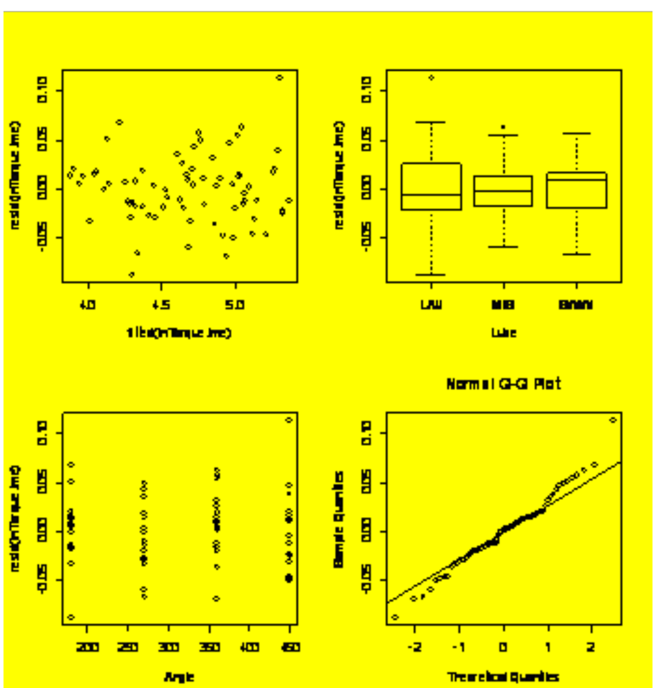



```
### Example 8.30 (p. 331) Torque = f(Lube,Unit(Lube),Angle) by general linear model.
Unit=gl(6,4,72)
Angle=rep(c(180,270,360,450),18,each=1)
Lube=rep(c("LAU","MIS","SWW"),each=24)
Torque=c(72.1,103.6,129.9,173.9,77.2,122.6,162.9,210.1,61.1,88.9,130.3,157.8,75.8,116.4,153,
198.4,67.3,105.4,154.1,222.5,70.6,107.6,144.9,197.3,70.6,102.1,145.7,193.3,65.2,91.9,123.7,
162.9,58.9,83.5,117.9,156.3,71.4,101.4,158.5,204.2,73.6,111.6,165.1,198.4,63.3,92.6,130.7,
168.7,53.4,80.2,112.4,138,49.4,70.6,100.7,135.4,53.4,80.2,122.2,153.7,50.5,72.8,106.1,129.6,
57.8,85.3,120.4,154.8,51.6,71.7,108.3,147.9)
Unit=factor(Unit)
Lube=factor(Lube)
lnTorque=log(Torque)
library(lattice)
xyplot(lnTorque~Angle|Lube)
```



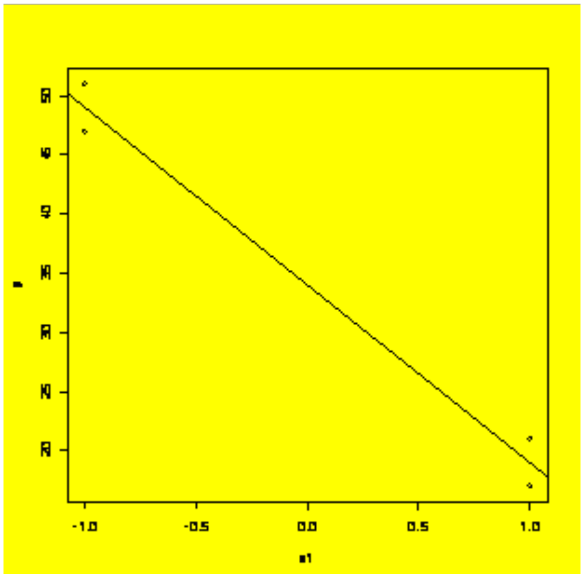

```
### DO NOT USE THE aov() SOLUTION. IT USES SEQUENTIAL INSTEAD OF ADJUSTED SUMS OF SQUARES!!!
### lnTorque.aov=aov(lnTorque~Lube*Angle+I(Angle^2)+Error(Lube/Unit))              #WRONG!!!
### summary(lnTorque.aov)
lnTorque.lme=lme(fixed=lnTorque~Lube+Angle+Lube:Angle+I(Angle^2),random=~1|Lube/Unit)
summary(lnTorque.lme)

Linear mixed-effects model fit by REML
```

```
 Data: NULL
        AIC       BIC   logLik
  -96.83632 -75.09245 58.41816

Random effects:
 Formula: ~1 | Lube
        (Intercept)
StdDev:  0.02239666

 Formula: ~1 | Unit %in% Lube
        (Intercept)   Residual
StdDev:  0.08822825 0.04114529

Fixed effects: lnTorque ~ Lube + Angle + Lube:Angle + I(Angle^2)
                  Value  Std.Error DF  t-value p-value
(Intercept)    3.284906 0.07352464 50 44.67762  0.0000
LubeMIS       -0.068217 0.07156538  0 -0.95321     NaN
LubeSWW       -0.316573 0.07156538  0 -4.42355     NaN
Angle          0.006128 0.00038627 50 15.86412  0.0000
I(Angle^2)    -0.000004 0.00000060 50 -6.48073  0.0000
LubeMIS:Angle  0.000010 0.00011804 50  0.08366  0.9337
LubeSWW:Angle  0.000063 0.00011804 50  0.53705  0.5936
 Correlation:
              (Intr) LubMIS LubSWW Angle  I(A^2) LMIS:A
LubeMIS       -0.487
LubeSWW       -0.487  0.500
Angle         -0.786  0.079  0.079
I(Angle^2)     0.725  0.000  0.000 -0.976
LubeMIS:Angle  0.253 -0.520 -0.260 -0.153  0.000
LubeSWW:Angle  0.253 -0.260 -0.520 -0.153  0.000  0.500

Standardized Within-Group Residuals:
        Min          Q1         Med          Q3         Max
-2.12910987 -0.46837120  0.06757591  0.43190371  2.76024297

Number of Observations: 72
Number of Groups:
          Lube Unit %in% Lube
             3             18
Warning message:
NaNs produced in: pt(q, df, lower.tail, log.p)

par(mfrow=c(2,2))
plot(resid(lnTorque.lme)~fitted(lnTorque.lme))
plot(resid(lnTorque.lme)~Lube)
plot(resid(lnTorque.lme)~Angle)
qqnorm(resid(lnTorque.lme)); qqline(resid(lnTorque.lme))
```

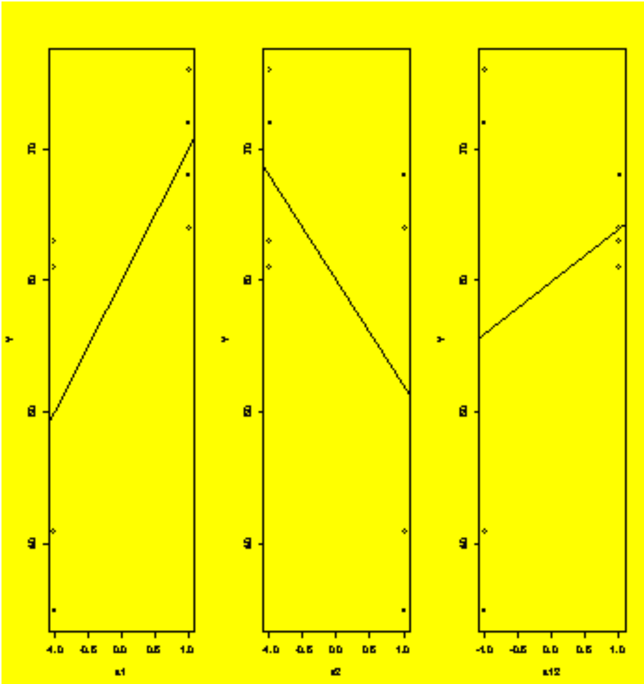



```
####################################################################################################

#              CHAPTER 9: Two-Level Factorial Experiments
```

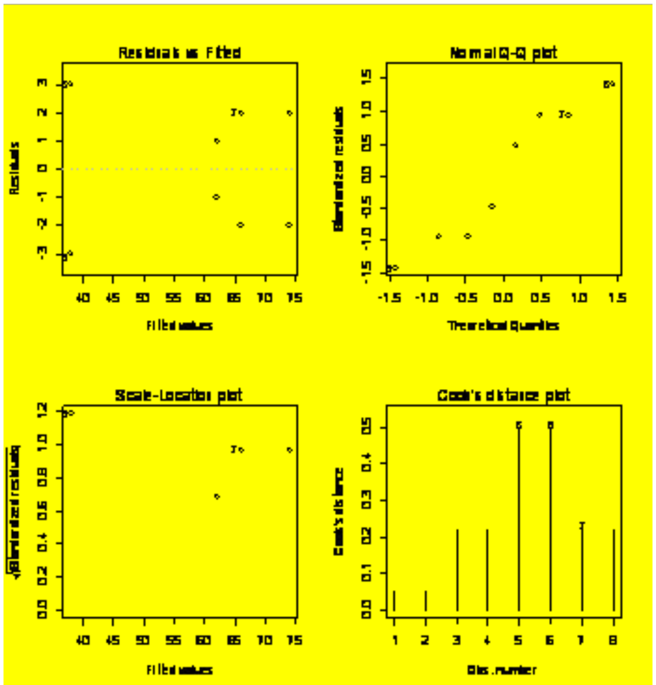
```
####################################################################################################

### Example 9.2 (p. 351) Analysis of a 2^1 experiment.
x1=c(-1,-1,1,1)
y=c(47,51,21,17)
y.fit=lm(y~x1)
plot(y~x1);abline(y.fit)
```



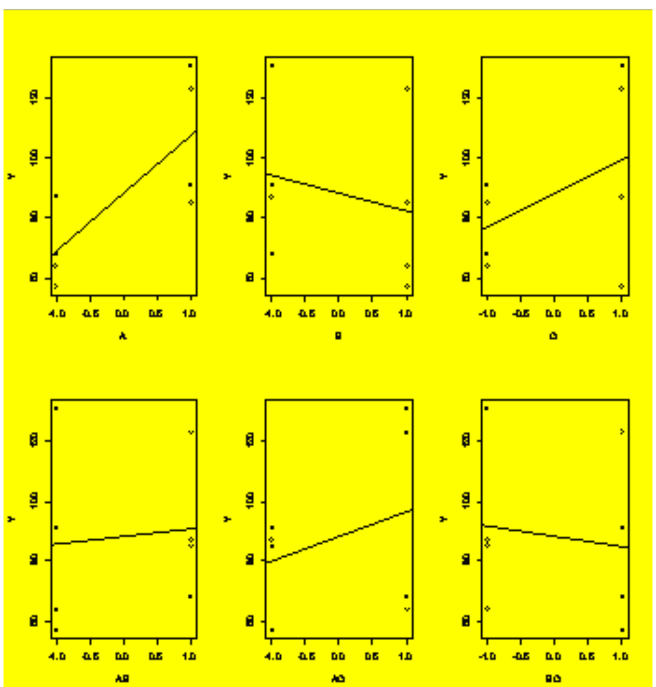

```
summary(y.fit)

Call:
lm(formula = y ~ x1)

Residuals:
 1  2  3  4 
-2  2  2 -2 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)   
(Intercept)   34.000      1.414   24.04  0.00173 **
x1           -15.000      1.414  -10.61  0.00877 **
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1 

Residual standard error: 2.828 on 2 degrees of freedom
Multiple R-Squared: 0.9825,     Adjusted R-squared: 0.9738 
F-statistic: 112.5 on 1 and 2 DF,  p-value: 0.008772 

anova(y.fit)

Analysis of Variance Table

Response: y
          Df Sum Sq Mean Sq F value   Pr(>F)   
x1         1    900     900   112.5 0.008772 **
Residuals  2     16       8                    
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1 


### Example 9.6 (p. 362) Analysis of a 2^2 experiment with two replicates.
x1=c(-1,-1,1,1,-1,-1,1,1)
x2=c(-1,-1,-1,-1,1,1,1,1)
x12=x1*x2
Y=c(61,63,76,72,41,35,68,64)
Y.data=data.frame(Y,x1,x2)
par(mfrow=c(1,3))                                              #Plot the data
plot(Y~x1,pch=x2); abline(lm(Y~x1))
plot(Y~x2,pch=x1); abline(lm(Y~x2))
plot(Y~x12); abline(lm(Y~x12))
```



```
Y.fit=lm(Y~x1*x2,data=Y.data)                                          #linear model with main effects and interaction
summary(Y.fit)                                                         #Table of regression coeficients

Call:
lm(formula = Y ~ x1 * x2, data = Y.data)

Residuals:
 1  2  3  4  5  6  7  8 
-1  1  2 -2  3 -3  2 -2 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   60.000      1.061  56.569 5.85e-07 ***
x1            10.000      1.061   9.428 0.000706 ***
x2            -8.000      1.061  -7.542 0.001655 ** 
x1:x2          4.000      1.061   3.771 0.019584 *  
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1 

Residual standard error: 3 on 4 degrees of freedom
Multiple R-Squared: 0.9756,     Adjusted R-squared: 0.9573 
F-statistic: 53.33 on 3 and 4 DF,  p-value: 0.001106 

anova(Y.fit)#ANOVA table

Analysis of Variance Table

Response: Y
          Df Sum Sq Mean Sq F value    Pr(>F)    
x1         1    800     800  88.889 0.0007056 ***
x2         1    512     512  56.889 0.0016552 ** 
x1:x2      1    128     128  14.222 0.0195835 *  
Residuals  4     36       9                      
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1 

Y.diagnostics=data.frame(Y,x1,x2,residuals(Y.fit),predict(Y.fit))   #Collect up the data, residuals, and fits
Y.diagnostics

   Y x1 x2 residuals.Y.fit. predict.Y.fit.
1 61 -1 -1               -1             62
2 63 -1 -1                1             62
3 76  1 -1                2             74
4 72  1 -1               -2             74
5 41 -1  1                3             38
6 35 -1  1               -3             38
7 68  1  1                2             66
8 64  1  1               -2             66

par(mfrow=c(2,2))                                                      #Prep for 2x2 matrix of diagnostic plots
plot(Y.fit)                                                            #Default diagnostic plots
```



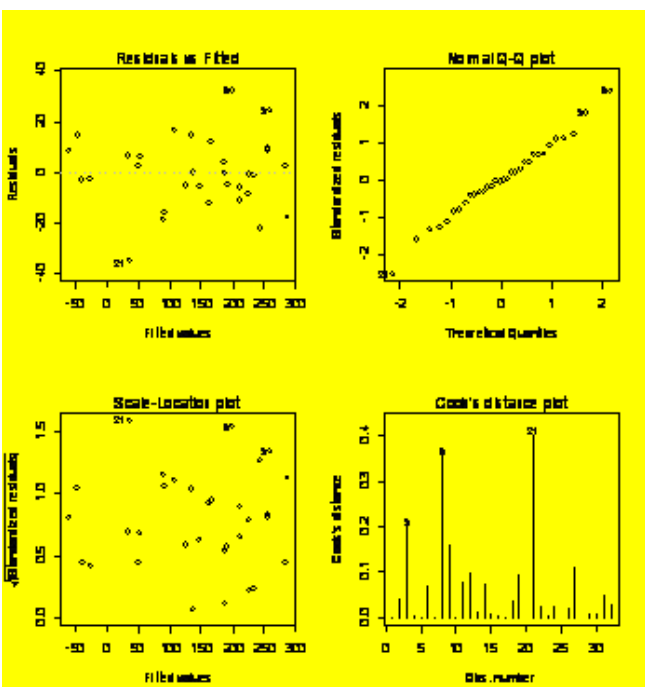

```
### Example 9.7 (p. 367) Refining the model for a 2^3 design.
A=c(1,1,-1,1,1,-1,-1,-1)
B=c(-1,1,-1,-1,1,-1,1,1)
C=c(-1,1,-1,1,-1,1,-1,1)
Y=c(91,123,68,131,85,87,64,57)
par(mfrow=c(2,3))                                                #Graphs: two rows, three columns
AB=A*B;AC=A*C;BC=B*C                                             #Interactions
plot(Y~A);abline(lm(Y~A));plot(Y~B);abline(lm(Y~B));plot(Y~C);abline(lm(Y~C))             #Plot the main effects
plot(Y~AB);abline(lm(Y~AB));plot(Y~AC);abline(lm(Y~AC));plot(Y~BC);abline(lm(Y~BC))       #... and the interactions
```



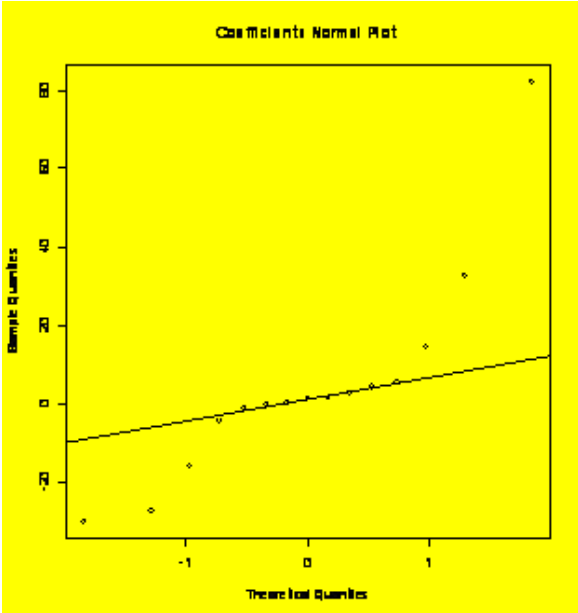
```
Y.fit.0=lm(Y~A*B*C)                                              #Fits the full model
summary(Y.fit.0)

Call:
lm(formula = Y ~ A * B * C)

Residuals:
ALL 8 residuals are 0: no residual degrees of freedom!

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
```

```
(Intercept)    88.25         NA      NA       NA
A              19.25         NA      NA       NA
B              -6.00         NA      NA       NA
C              11.25         NA      NA       NA
A:B             2.50         NA      NA       NA
A:C             8.25         NA      NA       NA
B:C            -3.50         NA      NA       NA
A:B:C           3.00         NA      NA       NA

Residual standard error: NaN on 0 degrees of freedom
Multiple R-Squared:     1,      Adjusted R-squared:   NaN
F-statistic:   NaN on 7 and 0 DF,  p-value: NA

anova(Y.fit.0)

Analysis of Variance Table

Response: Y
          Df Sum Sq Mean Sq F value Pr(>F)
A          1 2964.5  2964.5
B          1  288.0   288.0
C          1 1012.5  1012.5
A:B        1   50.0    50.0
A:C        1  544.5   544.5
B:C        1   98.0    98.0
A:B:C      1   72.0    72.0
Residuals  0    0.0

Y.fit.1=lm(Y~A*B*C-A:B:C)                                            #Drops the ABC interaction
Y.fit.2=lm(Y~A*B*C-A:B:C-A:B)
Y.fit.3=lm(Y~A*B*C-A:B:C-A:B-B:C)                                    #or equivalently: Y.fit.3=lm(Y~A+B+C+A:C)
Y.fit.4=lm(Y~A+C+A:C)
Y.fit.5=lm(Y~A+C)
Y.fit.6=lm(Y~A)
Y.fit.7=lm(Y~1)                                                      #Model constant (mean) only
anova(Y.fit.0,Y.fit.1,Y.fit.2,Y.fit.3,Y.fit.4,Y.fit.5,Y.fit.6,Y.fit.7)     #Compare all eight models

Analysis of Variance Table

Model 1: Y ~ A * B * C
Model 2: Y ~ A * B * C - A:B:C
Model 3: Y ~ A * B * C - A:B:C - A:B
Model 4: Y ~ A * B * C - A:B:C - A:B - B:C
Model 5: Y ~ A + C + A:C
Model 6: Y ~ A + C
Model 7: Y ~ A
Model 8: Y ~ 1
  Res.Df     RSS Df Sum of Sq F Pr(>F)
1      0     0.0
2      1    72.0 -1     -72.0
3      2   122.0 -1     -50.0
4      3   220.0 -1     -98.0
5      4   508.0 -1    -288.0
6      5  1052.5 -1    -544.5
7      6  2065.0 -1   -1012.5
8      7  5029.5 -1   -2964.5


### Example 9.10 (p. 379) Analysis of a 2^5 design.
Y=c(226,150,284,190,287,149,53,232,221,-30,76,270,59,-32,142,121,-43,200,123,137,1,
-51,187,265,233,217,71,187,207,40,179,266)
A=c(1,-1,-1,1,1,-1,1,1,-1,1,1,1,1,1,-1,1,-1,1,1,-1,-1,-1,1,-1,-1,-1,1,-1,1,-1,-1,-1)
B=c(1,1,1,1,1,-1,-1,1,1,-1,-1,1,-1,-1,-1,-1,-1,1,-1,-1,-1,-1,1,1,1,1,-1,1,1,-1,-1,1)
C=c(-1,1,1,1,1,-1,1,-1,-1,1,-1,1,1,1,-1,-1,1,1,-1,-1,1,1,-1,1,-1,-1,-1,1,-1,1,-1,-1)
D=c(-1,-1,1,-1,-1,-1,1,1,1,1,1,1,-1,-1,1,-1,1,1,1,1,-1,-1,-1,-1,1,-1,-1,1,1,1,-1,-1)
E=c(-1,1,-1,1,-1,1,-1,1,-1,1,1,-1,-1,1,-1,-1,1,1,-1,1,-1,1,1,-1,1,1,1,1,-1,-1,-1,-1)
Y.fit=lm(Y~A+B+C+D+E+A:B+A:C+A:D+A:E+B:C+B:D+B:E+C:D+C:E+D:E)
summary(Y.fit)

Call:
lm(formula = Y ~ A + B + C + D + E + A:B + A:C + A:D + A:E +
    B:C + B:D + B:E + C:D + C:E + D:E)

Residuals:
     Min       1Q   Median       3Q      Max
-34.5000  -6.4219  -0.4375   9.0625  32.5625

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 144.28125    3.39577  42.489  < 2e-16 ***
A            -4.28125    3.39577  -1.261 0.225471
B            82.09375    3.39577  24.175 5.05e-14 ***
C           -29.90625    3.39577  -8.807 1.56e-07 ***
D             1.46875    3.39577   0.433 0.671134
E           -27.21875    3.39577  -8.015 5.41e-07 ***
```

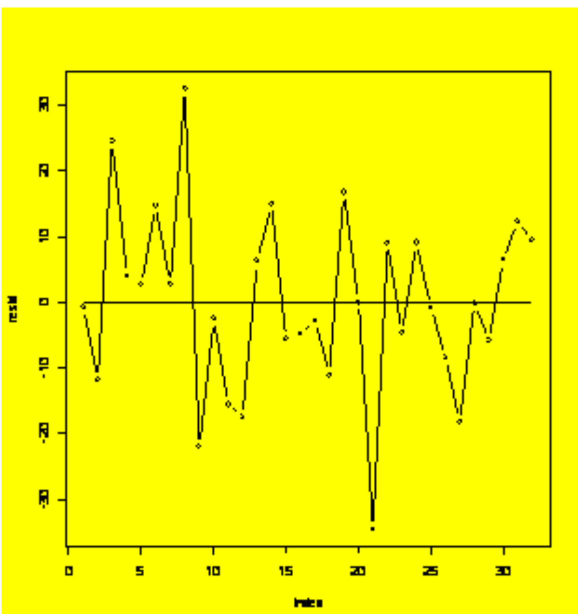
```
A:B           2.78125    3.39577   0.819 0.424799
A:C          14.53125    3.39577   4.279 0.000575 ***
A:D          -0.09375    3.39577  -0.028 0.978316
A:E          -1.03125    3.39577  -0.304 0.765280
B:C          32.65625    3.39577   9.617 4.72e-08 ***
B:D           1.40625    3.39577   0.414 0.684286
B:E           0.34375    3.39577   0.101 0.920626
C:D           4.28125    3.39577   1.261 0.225471
C:E         -15.78125    3.39577  -4.647 0.000268 ***
D:E           5.46875    3.39577   1.610 0.126847
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Residual standard error: 19.21 on 16 degrees of freedom
Multiple R-Squared: 0.9819,     Adjusted R-squared: 0.9648
F-statistic:  57.7 on 15 and 16 DF,  p-value: 4.702e-11

anova(Y.fit)

Analysis of Variance Table

Response: Y
          Df Sum Sq Mean Sq  F value    Pr(>F)
A          1    587     587   1.5895 0.2254709
B          1 215660  215660 584.4452 5.052e-14 ***
C          1  28620   28620  77.5617 1.560e-07 ***
D          1     69      69   0.1871 0.6711342
E          1  23708   23708  64.2481 5.408e-07 ***
A:B        1    248     248   0.6708 0.4247991
A:C        1   6757    6757  18.3117 0.0005750 ***
A:D        1 0.2812  0.2812   0.0008 0.9783163
A:E        1     34      34   0.0922 0.7652801
B:C        1  34126   34126  92.4818 4.718e-08 ***
B:D        1     63      63   0.1715 0.6842859
B:E        1      4       4   0.0102 0.9206265
C:D        1    587     587   1.5895 0.2254709
C:E        1   7970    7970  21.5976 0.0002683 ***
D:E        1    957     957   2.5936 0.1268468
Residuals 16   5904     369
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

par(mfrow=c(2,2))
plot(Y.fit)                                                   #Default residuals plots
```



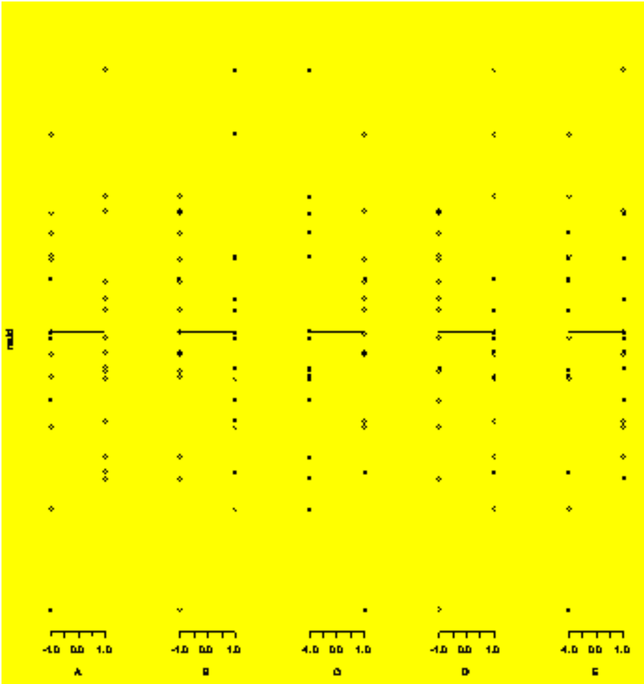

```
par(mfrow=c(1,1))
coeff=coefficients(Y.fit)[2:16]                               #The coefficients without the constant
qqnorm(coeff,main="Coefficients Normal Plot");qqline(coeff)   #Normal plot the coefficients
```



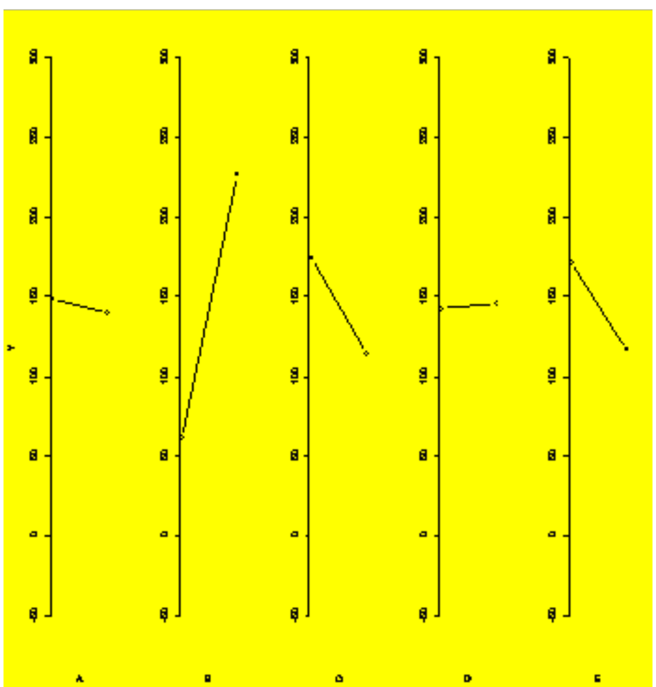

```
resid=residuals(Y.fit)
plot(resid,type="b");xref=c(1,length(resid));yref=c(0,0);lines(xref,yref)  #Residuals run chart
```




```
old.par = par(no.readonly = TRUE)
par(mfrow=c(1,5),bty="n",yaxt="n")                          #Five plots in one row
plot(resid~A);lines(c(-1,1),c(0,0))                         #Residuals versus A
plot(resid~B,ylab="");lines(c(-1,1),c(0,0))
plot(resid~C,ylab="");lines(c(-1,1),c(0,0))
plot(resid~D,ylab="");lines(c(-1,1),c(0,0))
plot(resid~E,ylab="");lines(c(-1,1),c(0,0))
```



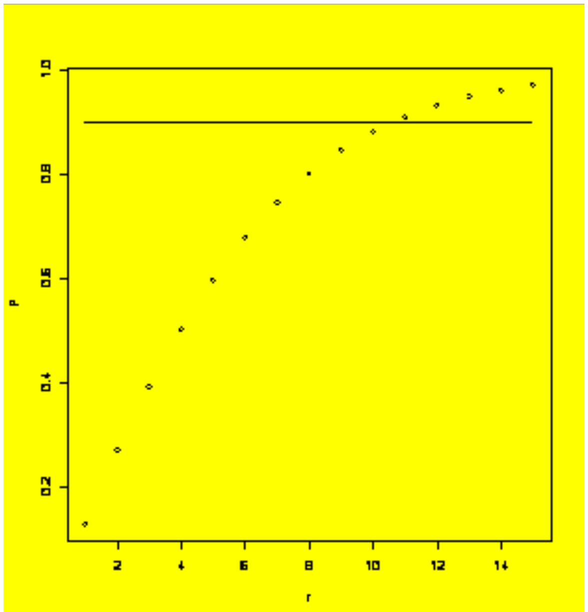
```
par(old.par)
par(mfrow=c(1,5),xaxt="n",bty="n")                                        #Five plots in one row
plot(aggregate(Y,list(A),mean),type="b",xlab="A",ylab="Y",ylim=c(min(Y),max(Y)))   #Main effects plots
plot(aggregate(Y,list(B),mean),type="b",xlab="B",ylab="",ylim=c(min(Y),max(Y)))
plot(aggregate(Y,list(C),mean),type="b",xlab="C",ylab="",ylim=c(min(Y),max(Y)))
plot(aggregate(Y,list(D),mean),type="b",xlab="D",ylab="",ylim=c(min(Y),max(Y)))
plot(aggregate(Y,list(E),mean),type="b",xlab="E",ylab="",ylim=c(min(Y),max(Y)))
```



```
par(old.par)                                          #Restore old graphics parameters


### Example 9.12 (p. 393) Power calculation for a 2^k design.
power=function(k,r,delta,sigma)                       #Find the power for a 2^k design with r replicates
{
N=r*2^k                                               #Total number of runs
lambda=N/2/2*(delta/sigma)^2                          #Noncentrality parameter
dfmodel=k+k*(k-1)/2                                   #Main effects and two factor interactions only
dferror=N-1-dfmodel                                   #Errror degrees of freedom
Falpha=qf(0.95,1,dferror)                             #F(alpha=0.05) assumed
pf(Falpha,1,dferror,lambda,lower.tail=FALSE)          #The power to detect effect delta
}
power(4,6,400,800)                                    #Gives the answer to the example
```

```
[1] 0.677884
```

```
### Example 9.13 (p. 394) Sample size calculation for a 2^k design.
r=c(1:15)                                              #Guess that the required r is in this range
P=power(4,r,400,800)                                   #Find the powers associated with r
plot(P~r);lines(c(1,15),c(0.90,0.90))                  #Use plot to find min r that gives power > 0.90
```




```
power(4,11,400,800)                                    #Exact power for r=11 replicates
```

```
[1] 0.909437
```

```
### Example 9.17 (p. 397) Find the number of replicates to quantify a coefficient.
replicates=function(k,delta,sigma)                     #k is the number of variables in the 2^k experiment
{
dfmodel=k+k*(k-1)/2                                    #Main effects and two-factor interactions
RHS=(1.96*sigma/delta)^2/2^k                           #For priming the loop, will always give low r
r=trunc(RHS)                                           #Conservative integer starting point
while (r<RHS)                                          #while (r is too small)
   {
   r=r+1                                               #Increment r
   dferror=r*2^k-1-dfmodel                             #New value
   RHS=(-qt(0.025,dferror)*sigma/delta)^2/2^k          #Assumes alpha = 0.05
   }
r                                                      #Report the result
}
replicates(3,20,80)                                    #The answer in the book, r=8, is just barely small
                                                       #because (r = 8) < (RHS = 8.08). The right answer is
                                                       #r=9, as this function confirms.
```

```
[1] 9
```

```
########################################################################################################################
### The function TLFF() below creates two-level balanced full factorial 2^k experiment designs for 2 to 7 factors with
### two-factor interactions. Use rbind() to replicate the design and use subset() to create the fractional factorial
### designs.

### Example: Create and analyze a 2^4 full factorial design with three replicates.
des.mat=TLFF(4)                                                        #Create the 2^4 full-factorial design
des.mat
```

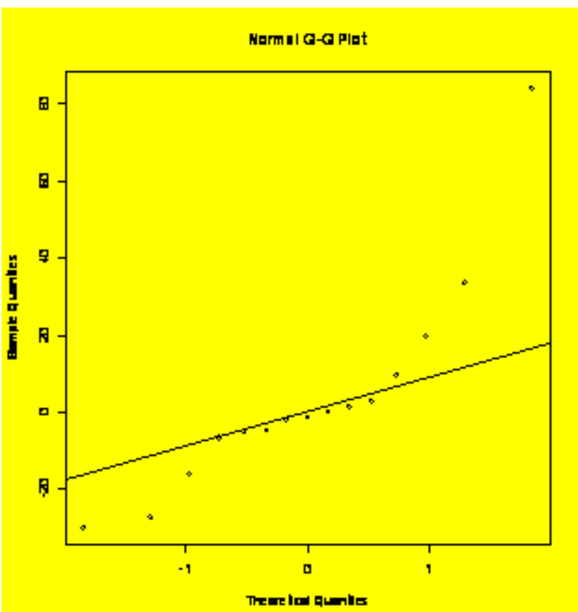
```
    A  B  C  D AB AC AD BC BD CD
1  -1 -1 -1 -1  1  1  1  1  1  1
2  -1 -1 -1  1  1  1 -1  1 -1 -1
3  -1 -1  1 -1  1 -1  1 -1  1 -1
4  -1 -1  1  1  1 -1 -1 -1 -1  1
5  -1  1 -1 -1 -1  1  1 -1 -1  1
6  -1  1 -1  1 -1  1 -1 -1  1 -1
7  -1  1  1 -1 -1 -1  1  1 -1 -1
8  -1  1  1  1 -1 -1 -1  1  1  1
9   1 -1 -1 -1 -1 -1 -1  1  1  1
10  1 -1 -1  1 -1 -1  1  1 -1 -1
11  1 -1  1 -1 -1  1 -1 -1  1 -1
12  1 -1  1  1 -1  1  1 -1 -1  1
```

```
13  1  1 -1 -1  1 -1 -1 -1 -1  1
14  1  1 -1  1  1 -1  1 -1  1 -1
15  1  1  1 -1  1  1 -1  1 -1 -1
16  1  1  1  1  1  1  1  1  1  1

cor(des.mat)                                                    #Check the correlation matrix

   A B C D AB AC AD BC BD CD
A  1 0 0 0  0  0  0  0  0  0
B  0 1 0 0  0  0  0  0  0  0
C  0 0 1 0  0  0  0  0  0  0
D  0 0 0 1  0  0  0  0  0  0
AB 0 0 0 0  1  0  0  0  0  0
AC 0 0 0 0  0  1  0  0  0  0
AD 0 0 0 0  0  0  1  0  0  0
BC 0 0 0 0  0  0  0  1  0  0
BD 0 0 0 0  0  0  0  0  1  0
CD 0 0 0 0  0  0  0  0  0  1

des.mat=rbind(des.mat,des.mat,des.mat)                          #Three replicates
Block=gl(3,16,48)                                               #Block identifier
Y=rnorm(48)                                                     #Create a column of response data
Y.des.mat=cbind(des.mat,Block,Y)                                #Bind the design matrix, block identifier, and
response
Y.fit=lm(Y~Block+A*B*C,data=Y.des.mat)                          #Create the model
summary(Y.fit)

Call:
lm(formula = Y ~ Block + A * B * C, data = Y.des.mat)

Residuals:
     Min       1Q   Median       3Q      Max
-1.49864 -0.56533 -0.03205  0.50933  1.54858

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.420217   0.199397  -2.107   0.0417 *
Block2       0.363708   0.281991   1.290   0.2049
Block3       0.312395   0.281991   1.108   0.2749
A           -0.165889   0.115122  -1.441   0.1578
B           -0.006415   0.115122  -0.056   0.9559
C            0.064719   0.115122   0.562   0.5773
A:B         -0.291043   0.115122  -2.528   0.0157 *
A:C         -0.220582   0.115122  -1.916   0.0629 .
B:C          0.034265   0.115122   0.298   0.7676
A:B:C       -0.171697   0.115122  -1.491   0.1441
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Residual standard error: 0.7976 on 38 degrees of freedom
Multiple R-Squared: 0.3056,     Adjusted R-squared: 0.1411
F-statistic: 1.858 on 9 and 38 DF,  p-value: 0.08914


#########################################################################################################################
TLFF = function(k)
#The function TLFF() creates two-level balanced full factorial 2^k experiment designs for 2 to 7 factors with
#two-factor interactions. Use rbind() to replicate the design and use subset() to create the fractional factorial
#designs. See also ffDesMatrix(BHH2) and ffFullMatrix(BHH2).
#By PGMathews, 21March05, paul@mmbstatistical.com.
{
if (k<2 || k>7) print("Error: k out of range.");return
N=2^k                                            #Number of runs: N = 2^k
A=rep(c(-1,1),1,each=N/2)                        #N/2 -1's followed by N/2 1's
B=rep(c(-1,1),2,each=N/4)
AB=A*B                                           #AB interaction
design.matrix=data.frame(A,B,AB)                 #Combine in a data.frame
if (k>2)                                         #Then add third variable (C)
   {
   C=rep(c(-1,1),4,each=N/8)
   AC=A*C; BC=B*C
   design.matrix=data.frame(A,B,C,AB,AC,BC)
   }
if (k>3)                                         #Then add fourth variable (D)
   {
   D=rep(c(-1,1),8,each=N/16)
   AD=A*D; BD=B*D; CD=C*D
   design.matrix=data.frame(A,B,C,D,AB,AC,AD,BC,BD,CD)
   }
if (k>4)                                         #Then add fifth variable (E)
   {
   E=rep(c(-1,1),16,each=N/32)
   AE=A*E; BE=B*E; CE=C*E; DE=D*E
   design.matrix=data.frame(A,B,C,D,E,AB,AC,AD,AE,BC,BD,BE,CD,CE,DE)
   }
```

```
if (k>5)                                                    #Then add sixth variable (F)
   {
   F=rep(c(-1,1),32,each=N/64)
   AF=A*F;BF=B*F;CF=C*F;DF=D*F;EF=E*F
   design.matrix=data.frame(A,B,C,D,E,F,AB,AC,AD,AE,AF,BC,BD,BE,BF,CD,CE,CF,DE,DF,EF)
   }
if (k>6)                                                    #Then add seventh variable (G)
   {
   G=rep(c(-1,1),64,each=N/128)
   AG=A*G;BG=B*G;CG=C*G;DG=D*G;EG=E*G;FG=F*G
   design.matrix=data.frame(A,B,C,D,E,F,G,AB,AC,AD,AE,AF,AG,BC,BD,BE,BF,BG,CD,CE,CF,CG,DE,DF,DG,EF,EG,FG)
   }
design.matrix
}                                                           #End function
#############################################################################################################
```

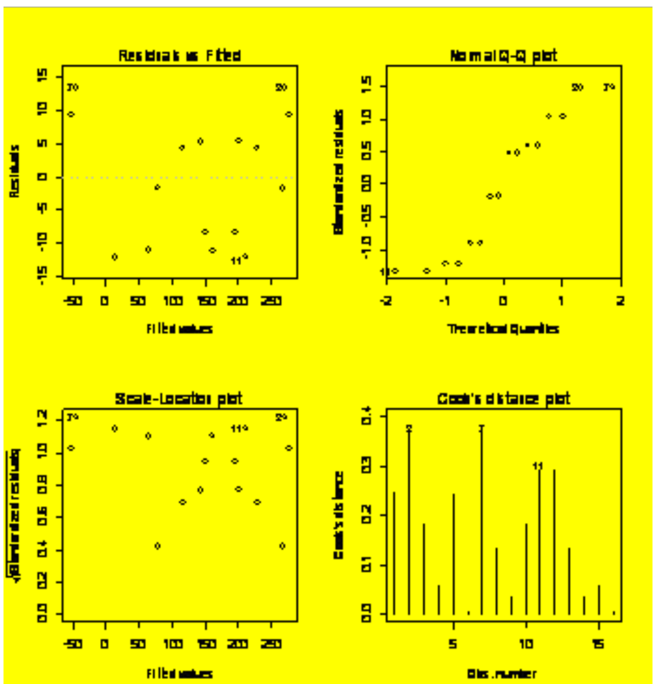
```
#############################################################################################################

#              CHAPTER 10: Fractional-Factorial Designs

#############################################################################################################

### Example 10.5 (p. 419) Analysis of a 2^(5-1) half-fractional factorial experiment.
### Start with the data from Example 9.10:
Y=c(226,150,284,190,287,149,53,232,221,-30,76,270,59,-32,142,121,-43,200,123,137,1,
-51,187,265,233,217,71,187,207,40,179,266)
A=c(1,-1,-1,1,1,-1,1,1,-1,1,1,1,1,1,-1,1,-1,1,1,-1,-1,-1,1,-1,-1,-1,1,-1,1,-1,-1,-1)
B=c(1,1,1,1,1,-1,-1,1,1,-1,-1,1,-1,-1,-1,-1,-1,1,-1,-1,-1,-1,1,1,1,1,1,-1,1,1,-1,-1,1)
C=c(-1,1,1,1,1,-1,1,-1,-1,1,-1,1,1,1,-1,-1,1,1,-1,-1,1,1,-1,1,-1,-1,-1,1,-1,1,-1,-1)
D=c(-1,-1,1,-1,-1,-1,1,1,1,1,1,1,-1,-1,1,-1,1,1,1,1,-1,-1,-1,-1,1,-1,-1,1,1,1,-1,-1)
E=c(-1,1,-1,1,-1,1,-1,1,-1,1,1,-1,-1,1,-1,-1,1,1,-1,1,-1,1,1,-1,1,1,1,1,-1,-1,-1,-1)
Y.full.factorial=data.frame(Y,A,B,C,D,E)                                      #Catch all of the data
rm(Y,A,B,C,D,E)                                                               #Clean up
Y.half.fraction=subset(Y.full.factorial,(A*B*C*D==E))                         #Create the subset
attach(Y.half.fraction)
AB=A*B;AC=A*C;AD=A*D;AE=A*E;BC=B*C;BD=B*D;BE=B*E;CD=C*D;CE=C*E;DE=D*E         #Make the interactions
Terms=data.frame(A,B,C,D,E,AB,AC,AD,AE,BC,BD,BE,CD,CE,DE)
cor(Terms)

   A B C D E AB AC AD AE BC BD BE CD CE DE
A  1 0 0 0 0  0  0  0  0  0  0  0  0  0  0
B  0 1 0 0 0  0  0  0  0  0  0  0  0  0  0
C  0 0 1 0 0  0  0  0  0  0  0  0  0  0  0
D  0 0 0 1 0  0  0  0  0  0  0  0  0  0  0
E  0 0 0 0 1  0  0  0  0  0  0  0  0  0  0
AB 0 0 0 0 0  1  0  0  0  0  0  0  0  0  0
AC 0 0 0 0 0  0  1  0  0  0  0  0  0  0  0
AD 0 0 0 0 0  0  0  1  0  0  0  0  0  0  0
AE 0 0 0 0 0  0  0  0  1  0  0  0  0  0  0
BC 0 0 0 0 0  0  0  0  0  1  0  0  0  0  0
BD 0 0 0 0 0  0  0  0  0  0  1  0  0  0  0
BE 0 0 0 0 0  0  0  0  0  0  0  1  0  0  0
CD 0 0 0 0 0  0  0  0  0  0  0  0  1  0  0
CE 0 0 0 0 0  0  0  0  0  0  0  0  0  1  0
DE 0 0 0 0 0  0  0  0  0  0  0  0  0  0  1

Y.fit=lm(Y~A+B+C+D+E+AB+AC+AD+AE+BC+BD+BE+CD+CE+DE)
summary(Y.fit)

Call:
lm(formula = Y ~ A + B + C + D + E + AB + AC + AD + AE + BC +
    BD + BE + CD + CE + DE)

Residuals:
ALL 16 residuals are 0: no residual degrees of freedom!

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 142.5625         NA      NA       NA
A            -5.1875         NA      NA       NA
B            84.1875         NA      NA       NA
C           -30.0625         NA      NA       NA
D             1.4375         NA      NA       NA
E           -27.5625         NA      NA       NA
AB           -1.3125         NA      NA       NA
AC           19.6875         NA      NA       NA
AD           -4.8125         NA      NA       NA
AE           -2.0625         NA      NA       NA
BC           33.5625         NA      NA       NA
BD            2.8125         NA      NA       NA
BE           -6.6875         NA      NA       NA
CD            9.5625         NA      NA       NA
CE          -16.1875         NA      NA       NA
```

```
DE            0.0625         NA      NA       NA

Residual standard error: NaN on 0 degrees of freedom
Multiple R-Squared:     1,      Adjusted R-squared:   NaN
F-statistic:   NaN on 15 and 0 DF,  p-value: NA

anova(Y.fit)

Analysis of Variance Table

Response: Y
          Df Sum Sq Mean Sq F value Pr(>F)
A          1    431     431
B          1 113401  113401
C          1  14460   14460
D          1     33      33
E          1  12155   12155
AB         1     28      28
AC         1   6202    6202
AD         1    371     371
AE         1     68      68
BC         1  18023   18023
BD         1    127     127
BE         1    716     716
CD         1   1463    1463
CE         1   4193    4193
DE         1 0.0625  0.0625
Residuals  0      0

coeff=coefficients(Y.fit)[2:16]                                            #The coefficients without the constant
qqnorm(coeff);qqline(coeff)                                        #Normal plot the coefficients
```




```
Y.fit=lm(Y~A+B+C+D+E+AC+BC+CD+CE)
summary(Y.fit)

Call:
lm(formula = Y ~ A + B + C + D + E + AC + BC + CD + CE)

Residuals:
    Min      1Q  Median      3Q     Max
-12.000  -8.844   1.375   6.406  13.500

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  142.562      3.692  38.617 2.01e-08 ***
A             -5.188      3.692  -1.405 0.209576
B             84.188      3.692  22.804 4.66e-07 ***
C            -30.062      3.692  -8.143 0.000184 ***
D              1.437      3.692   0.389 0.710438
E            -27.562      3.692  -7.466 0.000298 ***
AC            19.687      3.692   5.333 0.001773 **
BC            33.562      3.692   9.091 9.94e-05 ***
CD             9.562      3.692   2.590 0.041198 *
CE           -16.187      3.692  -4.385 0.004644 **
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1
```

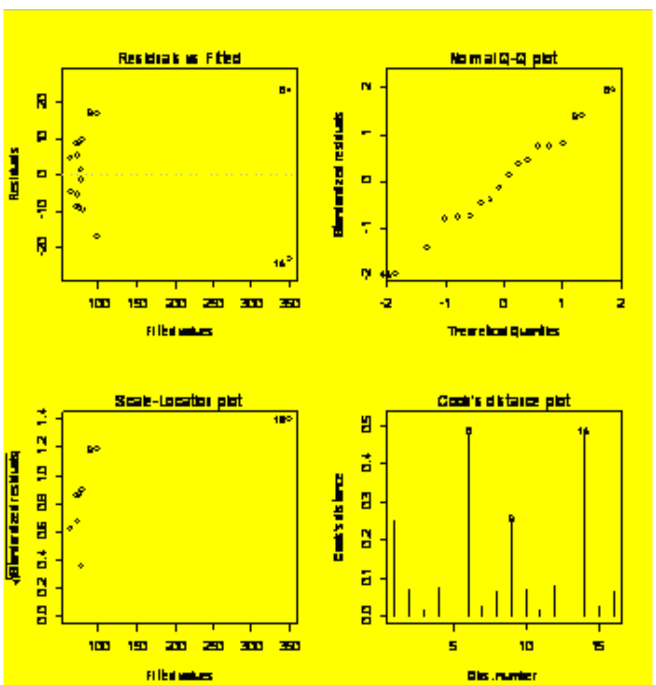
```
Residual standard error: 14.77 on 6 degrees of freedom
Multiple R-Squared: 0.9924,     Adjusted R-squared: 0.9809
F-statistic:  86.8 on 9 and 6 DF,  p-value: 1.163e-05

anova(Y.fit)

Analysis of Variance Table

Response: Y
          Df Sum Sq Mean Sq  F value    Pr(>F)
A          1    431     431   1.9745 0.2095758
B          1 113401  113401 520.0370 4.657e-07 ***
C          1  14460   14460  66.3116 0.0001844 ***
D          1     33      33   0.1516 0.7104376
E          1  12155   12155  55.7412 0.0002979 ***
AC         1   6202    6202  28.4394 0.0017733 **
BC         1  18023   18023  82.6509 9.945e-05 ***
CD         1   1463    1463   6.7094 0.0411983 *
CE         1   4193    4193  19.2264 0.0046440 **
Residuals  6   1308     218
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

par(mfrow=c(2,2))
plot(Y.fit)
```




```
### Example 10.8 (p. 424) Analysis of NIST sonoluminescence screening experiment in seven variables.
x1=c(-1,1,-1,1,-1,1,-1,1,-1,1,-1,1,-1,1,-1,1)
x2=c(-1,-1,1,1,-1,-1,1,1,-1,-1,1,1,-1,-1,1,1)
x3=c(-1,-1,-1,-1,1,1,1,1,-1,-1,-1,-1,1,1,1,1)
x4=c(-1,-1,-1,-1,-1,-1,-1,-1,1,1,1,1,1,1,1,1)
x5=c(-1,-1,1,1,1,1,-1,-1,1,1,-1,-1,-1,-1,1,1)
x6=c(-1,1,-1,1,1,-1,1,-1,1,-1,1,-1,-1,1,-1,1)
x7=c(-1,1,1,-1,1,-1,-1,1,-1,1,1,-1,1,-1,-1,1)
Y=c(80.6,66.1,59.1,68.9,75.1,373.8,66.8,79.6,114.3,84.1,68.4,88.1,78.1,327.2,77.6,61.9)
x12=x1*x2;x13=x1*x3;x14=x1*x4;x15=x1*x5;x16=x1*x6;x17=x1*x7         #Create the interactions
x23=x2*x3;x24=x2*x4;x25=x2*x5;x26=x2*x6;x27=x2*x7
x34=x3*x4;x35=x3*x5;x36=x3*x6;x37=x3*x7
x45=x4*x5;x46=x4*x6;x47=x4*x7
x56=x5*x6;x57=x5*x7
x67=x6*x7
X=data.frame(x1,x2,x3,x4,x5,x6,x7,x12,x13,x14,x15,x16,x17,x23,x24,x25,x26,x27,x34,x35,x36,x37,x45,x46,x47,x56,x57,x67)
cor(X)                                                                #Correlation matrix

    x1 x2 x3 x4 x5 x6 x7 x12 x13 x14 x15 x16 x17 x23 x24 x25 x26 x27 x34 x35 x36 x37 x45 x46 x47 x56 x57 x67
x1   1  0  0  0  0  0  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
x2   0  1  0  0  0  0  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
x3   0  0  1  0  0  0  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
x4   0  0  0  1  0  0  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
x5   0  0  0  0  1  0  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
x6   0  0  0  0  0  1  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
x7   0  0  0  0  0  0  1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
x12  0  0  0  0  0  0  0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   1   0   0
```

```
x13  0 0 0 0 0 0 0  0 1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0
x14  0 0 0 0 0 0 0  0 0 1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 1 0
x15  0 0 0 0 0 0 0  0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0
x16  0 0 0 0 0 0 0  0 0 0 0 1 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0
x17  0 0 0 0 0 0 0  0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 0 0 0 0 0
x23  0 0 0 0 0 0 0  0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 0 0 0 0 0
x24  0 0 0 0 0 0 0  0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 0 1
x25  0 0 0 0 0 0 0  0 0 0 0 1 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0
x26  0 0 0 0 0 0 0  0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0
x27  0 0 0 0 0 0 0  0 1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0
x34  0 0 0 0 0 0 0  0 0 0 0 1 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0
x35  0 0 0 0 0 0 0  0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 0 1
x36  0 0 0 0 0 0 0  0 0 1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 1 0
x37  0 0 0 0 0 0 0  1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 1 0 0
x45  0 0 0 0 0 0 0  0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 0 0 0 0 0
x46  0 0 0 0 0 0 0  0 1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0
x47  0 0 0 0 0 0 0  0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0
x56  0 0 0 0 0 0 0  1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 1 0 0
x57  0 0 0 0 0 0 0  0 0 1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 1 0
x67  0 0 0 0 0 0 0  0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 0 1

Y.fit=lm(Y~x1+x2+x3+x4+x5+x6+x7+x12+x13+x14+x15+x16+x17+x24)                #All other terms are confounded
summary(Y.fit)

Call:
lm(formula = Y ~ x1 + x2 + x3 + x4 + x5 + x6 + x7 + x12 + x13 +
    x14 + x15 + x16 + x17 + x24)

Residuals:
     1      2      3      4      5      6      7      8      9     10     11     12     13     14     15     16
-2.919  2.919  2.919 -2.919 -2.919  2.919  2.919 -2.919  2.919 -2.919 -2.919  2.919  2.919 -2.919 -2.919  2.919

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 110.6062     2.9187  37.895   0.0168 *
x1           33.1063     2.9187  11.343   0.0560 .
x2          -39.3062     2.9187 -13.467   0.0472 *
x3           31.9063     2.9187  10.931   0.0581 .
x4            1.8562     2.9187   0.636   0.6394
x5            3.7438     2.9187   1.283   0.4216
x6           -4.5188     2.9187  -1.548   0.3651
x7          -39.0562     2.9187 -13.381   0.0475 *
x12         -29.7812     2.9187 -10.203   0.0622 .
x13          35.0063     2.9187  11.994   0.0530 .
x14          -5.2437     2.9187  -1.797   0.3233
x15          -0.2813     2.9187  -0.096   0.9388
x16          -8.1688     2.9187  -2.799   0.2185
x17         -31.7313     2.9187 -10.872   0.0584 .
x24           0.8437     2.9187   0.289   0.8209
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Residual standard error: 11.67 on 1 degrees of freedom
Multiple R-Squared: 0.999,      Adjusted R-squared: 0.9849
F-statistic: 70.74 on 14 and 1 DF,  p-value: 0.09296

anova(Y.fit)

Analysis of Variance Table

Response: Y
          Df  Sum Sq Mean Sq  F value  Pr(>F)
x1         1 17536.4 17536.4 128.6549 0.05598 .
x2         1 24719.7 24719.7 181.3550 0.04719 *
x3         1 16288.1 16288.1 119.4972 0.05808 .
x4         1    55.1    55.1   0.4045 0.63939
x5         1   224.3   224.3   1.6452 0.42157
x6         1   326.7   326.7   2.3969 0.36510
x7         1 24406.3 24406.3 179.0553 0.04749 *
x12        1 14190.8 14190.8 104.1099 0.06219 .
x13        1 19607.0 19607.0 143.8459 0.05296 .
x14        1   440.0   440.0   3.2277 0.32334
x15        1     1.3     1.3   0.0093 0.93884
x16        1  1067.7  1067.7   7.8328 0.21847
x17        1 16110.0 16110.0 118.1900 0.05839 .
x24        1    11.4    11.4   0.0836 0.82085
Residuals  1   136.3   136.3
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Y.fit=lm(Y~x1+x2+x3+x7+x12+x13+x17)                         #These are, or are almost, significant
summary(Y.fit)

Call:
lm(formula = Y ~ x1 + x2 + x3 + x7 + x12 + x13 + x17)
```

```
Residuals:
       Min         1Q     Median         3Q        Max
-2.330e+01 -8.887e+00  3.331e-16  8.887e+00  2.330e+01

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  110.606      4.204  26.307 4.68e-09 ***
x1            33.106      4.204   7.874 4.89e-05 ***
x2           -39.306      4.204  -9.349 1.40e-05 ***
x3            31.906      4.204   7.589 6.37e-05 ***
x7           -39.056      4.204  -9.289 1.47e-05 ***
x12          -29.781      4.204  -7.083 0.000104 ***
x13           35.006      4.204   8.326 3.27e-05 ***
x17          -31.731      4.204  -7.547 6.63e-05 ***
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Residual standard error: 16.82 on 8 degrees of freedom
Multiple R-Squared: 0.9833,     Adjusted R-squared: 0.9686
F-statistic: 67.11 on 7 and 8 DF,  p-value: 1.784e-06

anova(Y.fit)

Analysis of Variance Table

Response: Y
          Df  Sum Sq Mean Sq F value    Pr(>F)
x1         1 17536.4 17536.4  62.003 4.894e-05 ***
x2         1 24719.7 24719.7  87.401 1.400e-05 ***
x3         1 16288.1 16288.1  57.590 6.372e-05 ***
x7         1 24406.3 24406.3  86.292 1.468e-05 ***
x12        1 14190.8 14190.8  50.174 0.0001037 ***
x13        1 19607.0 19607.0  69.324 3.272e-05 ***
x17        1 16110.0 16110.0  56.959 6.626e-05 ***
Residuals  8  2262.7   282.8
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

plot(Y.fit)
```

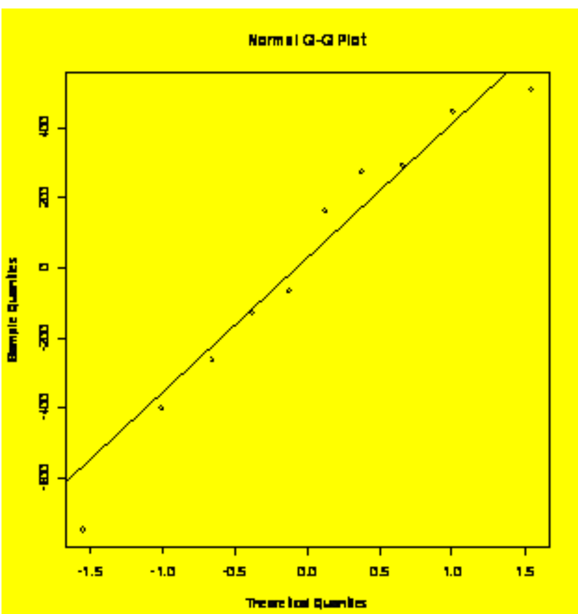



```
### Example 10.10 (p. 430) Creation of a resolution IV design by folding.
A=c(-1,-1,-1,-1,1,1,1,1)                                           #Base design: 2^3 in A, B, C
B=c(-1,-1,1,1,-1,-1,1,1)
C=c(-1,1,-1,1,-1,1,-1,1)
D=A*B;E=A*C;F=B*C;G=A*B*C                                          #Apply the generators
A=c(A,-A);B=c(B,-B);C=c(C,-C);D=c(D,-D);E=c(E,-E);F=c(F,-F);G=c(G,-G)     #Create the fold-over design
AB=A*B;AC=A*C;AD=A*D;AE=A*E;AF=A*F;AG=A*G                          #Create the interactions
BC=B*C;BD=B*D;BE=B*E;BF=B*F;BG=B*G
CD=C*D;CE=C*E;CF=C*F;CG=C*G
DE=D*E;DF=D*F;DG=D*G
EF=E*F;EG=E*G
FG=F*G
```

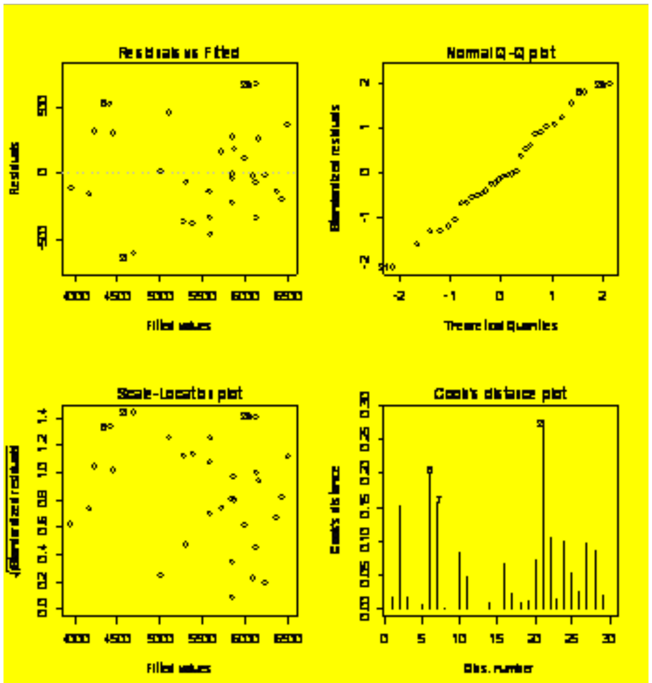
```
Terms=data.frame(A,B,C,D,E,F,G,AB,AC,AD,AE,AF,AG,BC,BD,BE,BF,BG,CD,CE,CF,CG,DE,DF,DG,EF,EG,FG)
cor(Terms)              #Inspection of the correlation matrix shows that the fold-over design is resolution IV.

   A B C D E F G AB AC AD AE AF AG BC BD BE BF BG CD CE CF CG DE DF DG EF EG FG
A  1 0 0 0 0 0 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
B  0 1 0 0 0 0 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
C  0 0 1 0 0 0 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
D  0 0 0 1 0 0 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
E  0 0 0 0 1 0 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
F  0 0 0 0 0 1 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
G  0 0 0 0 0 0 1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
AB 0 0 0 0 0 0 0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  1  0  0
AC 0 0 0 0 0 0 0  0  1  0  0  0  0  0  0  0  0  1  0  0  0  0  0  1  0  0  0  0
AD 0 0 0 0 0 0 0  0  0  1  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  1  0
AE 0 0 0 0 0 0 0  0  0  0  1  0  0  0  0  0  1  0  0  0  0  0  0  0  1  0  0  0
AF 0 0 0 0 0 0 0  0  0  0  0  1  0  0  0  1  0  0  1  0  0  0  0  0  0  0  0  0
AG 0 0 0 0 0 0 0  0  0  0  0  0  1  1  0  0  0  0  0  0  0  0  1  0  0  0  0  0
BC 0 0 0 0 0 0 0  0  0  0  0  0  1  1  0  0  0  0  0  0  0  0  1  0  0  0  0  0
BD 0 0 0 0 0 0 0  0  0  0  0  0  0  0  1  0  0  0  0  1  0  0  0  0  0  0  0  1
BE 0 0 0 0 0 0 0  0  0  0  0  1  0  0  0  1  0  0  1  0  0  0  0  0  0  0  0  0
BF 0 0 0 0 0 0 0  0  0  0  1  0  0  0  0  0  1  0  0  0  0  0  0  0  1  0  0  0
BG 0 0 0 0 0 0 0  0  1  0  0  0  0  0  0  0  0  1  0  0  0  0  0  1  0  0  0  0
CD 0 0 0 0 0 0 0  0  0  0  0  1  0  0  0  1  0  0  1  0  0  0  0  0  0  0  0  0
CE 0 0 0 0 0 0 0  0  0  0  0  0  0  0  1  0  0  0  0  1  0  0  0  0  0  0  0  1
CF 0 0 0 0 0 0 0  0  0  1  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  1  0
CG 0 0 0 0 0 0 0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  1  0  0
DE 0 0 0 0 0 0 0  0  0  0  0  0  1  1  0  0  0  0  0  0  0  0  1  0  0  0  0  0
DF 0 0 0 0 0 0 0  0  1  0  0  0  0  0  0  0  0  1  0  0  0  0  0  1  0  0  0  0
DG 0 0 0 0 0 0 0  0  0  0  1  0  0  0  0  0  1  0  0  0  0  0  0  0  1  0  0  0
EF 0 0 0 0 0 0 0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  1  0  0
EG 0 0 0 0 0 0 0  0  0  1  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  1  0
FG 0 0 0 0 0 0 0  0  0  0  0  0  0  0  1  0  0  0  0  1  0  0  0  0  0  0  0  1


### Example 10.11 (p. 433) Power calculation for a fractional factorial design with blocking.
### Start from the power function created in Example 9.12 and make appropriate modifications:
power=function(k,p,r,dfmodel,delta,sigma)           #Find the power for a 2^(k-p) design with r replicates in blocks
{
N=r*2^(k-p)                                         #Total number of runs
lambda=N/2/2*(delta/sigma)^2                        #Noncentrality parameter
dferror=N-1-dfmodel                                 #Errror degrees of freedom
Falpha=qf(0.95,1,dferror)                           #F(alpha=0.05) assumed
pf(Falpha,1,dferror,lambda,lower.tail=FALSE)        #The power to detect effect delta
}
power(5,2,4,10,100,80)                              #dfmodel =      5         +       2        +    3
                                                    #          (main effects) + (interactions) + (blocks)

[1] 0.9206987


### Example: Use the TLFF() function from Chapter 9 to create and analyze an experiment using two replicates of a
2^(7-4)
### sixteenth-fractional factorial design.
des.mat=TLFF(7)                                                  #Create the 2^7 full-factorial design
des.mat=subset(des.mat,(D==A*B & E==A*C & F==B*C & G==A*B*C))    #Use the generators to isolate the sixteenth
fraction
cor(des.mat)                                                     #Check the correlation matrix

   A B C D E F G AB AC AD AE AF AG BC BD BE BF BG CD CE CF CG DE DF DG EF EG FG
A  1 0 0 0 0 0 0  0  0  0  0  0  0  0  1  0  0  0  0  1  0  0  0  0  0  0  0  1
B  0 1 0 0 0 0 0  0  0  1  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  1  0
C  0 0 1 0 0 0 0  0  0  0  1  0  0  0  0  0  1  0  0  0  0  0  0  0  1  0  0  0
D  0 0 0 1 0 0 0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  1  0  0
E  0 0 0 0 1 0 0  0  1  0  0  0  0  0  0  0  0  1  0  0  0  0  0  1  0  0  0  0
F  0 0 0 0 0 1 0  0  0  0  0  0  1  1  0  0  0  0  0  0  0  0  1  0  0  0  0  0
G  0 0 0 0 0 0 1  0  0  0  0  1  0  0  0  1  0  0  1  0  0  0  0  0  0  0  0  0
AB 0 0 0 1 0 0 0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  1  0  0
AC 0 0 0 0 1 0 0  0  1  0  0  0  0  0  0  0  0  1  0  0  0  0  0  1  0  0  0  0
AD 0 1 0 0 0 0 0  0  0  1  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  1  0
AE 0 0 1 0 0 0 0  0  0  0  1  0  0  0  0  0  1  0  0  0  0  0  0  0  1  0  0  0
AF 0 0 0 0 0 0 1  0  0  0  0  1  0  0  0  1  0  0  1  0  0  0  0  0  0  0  0  0
AG 0 0 0 0 0 1 0  0  0  0  0  0  1  1  0  0  0  0  0  0  0  0  1  0  0  0  0  0
BC 0 0 0 0 0 1 0  0  0  0  0  0  1  1  0  0  0  0  0  0  0  0  1  0  0  0  0  0
BD 1 0 0 0 0 0 0  0  0  0  0  0  0  0  1  0  0  0  0  1  0  0  0  0  0  0  0  1
BE 0 0 0 0 0 0 1  0  0  0  0  1  0  0  0  1  0  0  1  0  0  0  0  0  0  0  0  0
BF 0 0 1 0 0 0 0  0  0  0  1  0  0  0  0  0  1  0  0  0  0  0  0  0  1  0  0  0
BG 0 0 0 0 1 0 0  0  1  0  0  0  0  0  0  0  0  1  0  0  0  0  0  1  0  0  0  0
CD 0 0 0 0 0 0 1  0  0  0  0  1  0  0  0  1  0  0  1  0  0  0  0  0  0  0  0  0
CE 1 0 0 0 0 0 0  0  0  0  0  0  0  0  1  0  0  0  0  1  0  0  0  0  0  0  0  1
CF 0 1 0 0 0 0 0  0  0  1  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  1  0
CG 0 0 0 1 0 0 0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  1  0  0
DE 0 0 0 0 0 1 0  0  0  0  0  0  1  1  0  0  0  0  0  0  0  0  1  0  0  0  0  0
DF 0 0 0 0 1 0 0  0  1  0  0  0  0  0  0  0  0  1  0  0  0  0  0  1  0  0  0  0
DG 0 0 1 0 0 0 0  0  0  0  1  0  0  0  0  0  1  0  0  0  0  0  0  0  1  0  0  0
EF 0 0 0 1 0 0 0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  1  0  0
```

```
EG 0 1 0 0 0 0 0  0  0  1  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  1  0
FG 1 0 0 0 0 0 0  0  0  0  0  0  0  0  1  0  0  0  0  1  0  0  0  0  0  0  0  1

des.mat=rbind(des.mat,des.mat)                                     #Two replicates
Block=gl(2,8,16)                                                   #Block identifier
Y=rnorm(16)                                                        #Create a column of response data
Y.des.mat=cbind(des.mat,Block,Y)                                   #Bind the design matrix, block identifier, and
response
Y.fit=lm(Y~Block+A+B+C+D+E+F+G,data=Y.des.mat)                     #Create the model
summary(Y.fit)

Call:
lm(formula = Y ~ Block + A + B + C + D + E + F + G, data = Y.des.mat)

Residuals:
       Min         1Q     Median         3Q        Max
-1.034e+00 -4.459e-01  1.388e-17  4.459e-01  1.034e+00

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.377205   0.335707  -1.124   0.2982
Block2       0.479596   0.474761   1.010   0.3460
A           -0.511302   0.237381  -2.154   0.0682 .
B            0.057109   0.237381   0.241   0.8168
C           -0.206127   0.237381  -0.868   0.4140
D           -0.233388   0.237381  -0.983   0.3583
E            0.284044   0.237381   1.197   0.2704
F           -0.007643   0.237381  -0.032   0.9752
G           -0.265588   0.237381  -1.119   0.3001
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Residual standard error: 0.9495 on 7 degrees of freedom
Multiple R-Squared: 0.5912,     Adjusted R-squared: 0.124
F-statistic: 1.265 on 8 and 7 DF,  p-value: 0.3846



##############################################################################################################

#              CHAPTER 11: Response Surface Designs

##############################################################################################################

### Example 11.9 (p. 460) Optimization of lamp lumens as a function of three geometry variables.
Lumens=c(4010,5135,5879,6073,3841,4933,5569,5239,5017,5243,6412,6210,5805,5624,5843,4746,6052,6105,6232,4549,
4080,5006,5438,4903,6129,6234,6860,6794,5780,6053)
A=c(-1,-1,1,1,-1,-1,1,1,0,0,0,0,0,0,0,-1,-1,1,1,-1,-1,1,1,0,0,0,0,0,0,0)
B=c(-1,1,-1,1,0,0,0,0,-1,-1,1,1,0,0,0,-1,1,-1,1,0,0,0,0,-1,-1,1,1,0,0,0)
C=c(0,0,0,0,-1,1,-1,1,-1,1,-1,1,0,0,0,0,0,0,0,-1,1,-1,1,-1,1,-1,1,0,0,0)
Block=gl(2,15,30)
Run=c(9,8,10,12,3,15,6,14,13,5,1,11,4,7,2,26,16,27,19,23,30,22,18,17,25,20,29,21,24,28)
AB=A*B;AC=A*C;BC=B*C;AA=A*A;BB=B*B;CC=C*C
Terms=data.frame(A,B,C,AB,AC,BC,AA,BB,CC)
cor(Terms)

   A B C AB AC BC          AA          BB          CC
A  1 0 0  0  0  0  0.00000000  0.00000000  0.00000000
B  0 1 0  0  0  0  0.00000000  0.00000000  0.00000000
C  0 0 1  0  0  0  0.00000000  0.00000000  0.00000000
AB 0 0 0  1  0  0  0.00000000  0.00000000  0.00000000
AC 0 0 0  0  1  0  0.00000000  0.00000000  0.00000000
BC 0 0 0  0  0  1  0.00000000  0.00000000  0.00000000
AA 0 0 0  0  0  0  1.00000000 -0.07142857 -0.07142857
BB 0 0 0  0  0  0 -0.07142857  1.00000000 -0.07142857
CC 0 0 0  0  0  0 -0.07142857 -0.07142857  1.00000000

Lumens.fit=lm(Lumens~Block+A+B+C+AB+AC+BC+AA+BB+CC)
summary(Lumens.fit)

Call:
lm(formula = Lumens ~ Block + A + B + C + AB + AC + BC + AA +
    BB + CC)

Residuals:
    Min      1Q  Median      3Q     Max
-604.97 -184.77  -27.95  240.71  673.23

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  5845.57     169.74  34.438  < 2e-16 ***
Block2        275.20     138.60   1.986 0.061697 .
A             512.19      94.89   5.398 3.30e-05 ***
B             448.50      94.89   4.727 0.000147 ***
C             162.56      94.89   1.713 0.102951
```

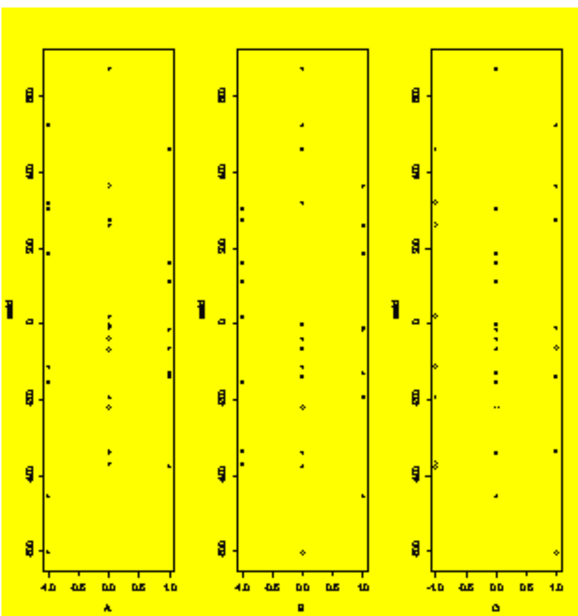
```
AB           -263.75     134.19  -1.965 0.064153 .
AC            -65.13     134.19  -0.485 0.633009
BC           -128.50     134.19  -0.958 0.350308
AA           -749.15     139.67  -5.364 3.55e-05 ***
BB            294.98     139.67   2.112 0.048163 *
CC           -402.15     139.67  -2.879 0.009608 **
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Residual standard error: 379.6 on 19 degrees of freedom
Multiple R-Squared: 0.8476,     Adjusted R-squared: 0.7673
F-statistic: 10.56 on 10 and 19 DF,  p-value: 8.155e-06

anova(Lumens.fit)

Analysis of Variance Table

Response: Lumens
          Df  Sum Sq Mean Sq F value    Pr(>F)
Block      1  568013  568013  3.9428 0.0616966 .
A          1 4197377 4197377 29.1354 3.296e-05 ***
B          1 3218436 3218436 22.3403 0.0001469 ***
C          1  422825  422825  2.9350 0.1029507
AB         1  556513  556513  3.8629 0.0641532 .
AC         1   33930   33930  0.2355 0.6330091
BC         1  132098  132098  0.9169 0.3503075
AA         1 4105241 4105241 28.4959 3.757e-05 ***
BB         1  789060  789060  5.4771 0.0303311 *
CC         1 1194249 1194249  8.2897 0.0096079 **
Residuals 19 2737225  144064
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

coeff=coefficients(Lumens.fit)[2:11]
qqnorm(coeff);qqline(coeff)
```




```
par(mfrow=c(2,2))
plot(Lumens.fit)
```




```
resid=residuals(Lumens.fit)
par(mfrow=c(1,3))
plot(resid~A);plot(resid~B);plot(resid~C)
```




```
##############################################################################################################

#              Appendices: Statistical Tables

###############################################################################################################

### Appendix A.2 (p. 478) Normal distribution.
pnorm(-1.96)                                  #cdf: P(-inf < z < -1.96) = 0.02499790

[1] 0.02499790

qnorm(0.025)                                  #inverse cdf: P(-inf < z < -1.959964) = 0.025

[1] -1.959964


### Appendix A.3 (p. 480) Student's t distribution.
pt(-2.5,23)                                   #cdf: P(-inf < t < -2.5;df = 23) = 0.01
```

```
[1] 0.009997061

qt(0.025,12)                              #inverse cdf: P(-inf < t < -2.179;df = 12) = 0.025

[1] -2.178813


### Appendix A.4 (p. 481) Chi-square distribution.
pchisq(8.0,4)                             #cdf: P(0 < X2 < 8.0;df=4) = 0.9084

[1] 0.9084218

qchisq(0.975,10)                          #inverse cdf: P(0 < X2 < 20.48;df = 10) = 0.975

[1] 20.48318


### Appendix A.5 (p. 482) F distribution.
pf(4.0,4,15)                              #cdf: P(0 < F < 4.0;dfnum=4,dfdenom=15) = 0.9790

[1] 0.978958

qf(0.95,4,15)                             #inverse cdf: P(0 < F < 3.056) = 0.95

[1] 3.055568


### Appendix A.6 (p. 484) Duncan's multiple range test.
### Not available?


### Appendix A.7 (p. 485) Studentized range distribution.
ptukey(4.020,4,17)                        #Inverse cdf: P(0 < Q < 4.020;k=4,df=17) = 0.95

[1] 0.950001

qtukey(0.95,4,17)                         #SRD cdf: P(0 < Q < 4.020;k=4,df=17) = 0.95

[1] 4.019985


### Appendix A.9 (p. 487) Fisher's Z transform.
FishersZ=function(r) log((1+r)/(1-r))/2   #Returns Z for a given r
FishersZ(0.98)                            #Fisher’s Z: Z(r = 0.98) = 2.29756

[1] 2.29756

invFishersZ=function(thisZ)               #Returns r for a given Z
{
r=-9999:9999
r=r/10000
Z=FishersZ(r)
thisr=approx(Z,r,xout=thisZ)
thisr$y
}
invFishersZ(2.29756)                      #Inverse Fisher’s Z: r(Z=2.29756) = 0.98

[1] 0.98
```